Student Handbook

Built-In Workbooks

Reference

Eclipse Studios

How to Use the Student Handbook

The Student Handbook is the additional skill and reference material found at the end of the text. This handbook can help you answer these questions.

What if I Forget What I Learned Last Year?

Use the **Prerequisite Skills** section to refresh your memory about things you have learned in other math classes. Here's a list of the topics covered in your book.

1. The FOIL Method
2. Factoring Polynomials
3. Congruent and Similar Figures
4. Pythagorean Theorem
5. Mean, Median, and Mode
6. Bar and Line Graphs
7. Frequency Tables and Histograms
8. Stem-and-Leaf Plots
9. Box-and-Whisker Plots

What If I Need More Practice?

You, or your teacher, may decide that working through some additional problems would be helpful. The **Extra Practice** section provides these problems for each lesson so you have ample opportunity to practice new skills.

What If I Have Trouble with Word Problems?

The **Mixed Problem Solving** portion of the book provides additional word problems that use the skills presented in each lesson. These problems give you real-world situations where math can be applied.

What If I Need to Practice for a Standardized Test?

You can review the types of problems commonly used for standardized tests in the **Preparing for Standardized Tests** section. This section includes examples and practice with multiple-choice, griddable or grid-in, and extended-response test items.

What If I Forget a Vocabulary Word?

The **English-Spanish Glossary** provides a list of important or difficult words used throughout the textbook. It provides a definition in English and Spanish as well as the page number(s) where the word can be found.

What If I Need to Check a Homework Answer?

The answers to odd-numbered problems are included in **Selected Answers**. Check your answers to make sure you understand how to solve all of the assigned problems.

What If I Need to Find Something Quickly?

The **Index** alphabetically lists the subjects covered throughout the entire textbook and the pages on which each subject can be found.

What if I Forget a Formula?

Inside the back cover of your math book is a list of **Formulas and Symbols** that are used in the book.

Prerequisite Skills

1 The FOIL Method

The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

EXAMPLE 1

Find $(x + 3)(x - 5)$.

F L I O

$$(x + 3)(x - 5) = \underbrace{x \cdot x}_{\text{First}} + \underbrace{(-5) \cdot x}_{\text{Outer}} + \underbrace{3 \cdot x}_{\text{Inner}} + \underbrace{(-3) \cdot 5}_{\text{Last}}$$

$$= x^2 - 5x + 3x - 15$$

$$= x^2 - 2x - 15$$

EXAMPLE 2

Find $(3y + 2)(5y + 4)$.

$$(3y + 2)(5y + 4) = y \cdot y + 4 \cdot 3y + 2 \cdot 5y + 2 \cdot 4$$

$$= y^2 + 12y + 10y + 8$$

$$= y^2 + 22y + 8$$

Exercises **Find each product.**

1. $(a + 2)(a + 4)$
2. $(v - 7)(v - 1)$
3. $(h + 4)(h - 4)$
4. $(d - 1)(d + 1)$
5. $(b + 4)(b - 3)$
6. $(s - 9)(s + 11)$
7. $(r + 3)(r - 8)$
8. $(k - 2)(k + 5)$
9. $(p + 8)(p + 8)$
10. $(x - 15)(x - 15)$
11. $(2c + 1)(c - 5)$
12. $(7n - 2)(n + 3)$
13. $(3m + 4)(2m - 5)$
14. $(5g + 1)(6g + 9)$
15. $(2q - 17)(q + 2)$
16. $(4t - 7)(3t - 12)$

NUMBER **For Exercises 17 and 18, use the following information.**

I'm thinking of two integers. One is 7 less than a number, and the other is 2 greater than the same number.

17. Write expressions for the two numbers.
18. Write a polynomial expression for the product of the numbers.

OFFICE SPACE **For Exercises 19–21, use the following information.**

Monica's current office is square. Her office in the company's new building will be 3 feet wider and 5 feet longer.

19. Write expressions for the dimensions of Monica's new office.
20. Write a polynomial expression for the area of Monica's new office.
21. Suppose Monica's current office is 7 feet by 7 feet. How much larger will her new office be?

2 Factoring Polynomials

Some polynomials can be factored using the Distributive Property.

EXAMPLE

1 Factor $4a^2 + 8a$.

Find the GCF of $4a^2$ and $8a$.

$4a^2 = 2 \cdot 2 \cdot a \cdot a$ $\quad$ $8a = 2 \cdot 2 \cdot 2 \cdot a$ $\quad$ GCF: $2 \cdot 2 \cdot a$ or $4a$

$4a^2 + 8a = 4a(a) + 4a(2)$ $\quad$ Rewrite each term using the GCF.

$= 4a(a + 2)$ $\quad$ Distributive Property

To factor quadratic trinomials of the form $x^2 + bx + c$, find two integers m and n with a product of c and with a sum of b. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

EXAMPLE

2 Factor each polynomial.

a. $x^2 + 5x + 6$ $\leftarrow$ Both b and c are positive.

In this trinomial, b is 5 and c is 6. Find two numbers with a product of 6 and a sum of 5.

Factors of 6	Sum of Factors
1, 6	7
2, 3	5

The correct factors are 2 and 3.

$x^2 + 5x + 6 = (x + m)(x + n)$ $\quad$ Write the pattern.

$= (x + 2)(x + 3)$ $\quad$ $m = 2$ and $n = 3$

CHECK Multiply the binomials to check the factorization.

$(x + 2)(x + 3) = x^2 + 3x + 2x + 2(3)$ $\quad$ FOIL

$= x^2 + 5x + 6$ ✓

b. $x^2 - 8x + 12$ $\leftarrow$ b is negative and c is positive.

In this trinomial, $b = -8$ and $c = 12$. This means that $m + n$ is negative and mn is positive. So m and n must both be negative.

Factors of 12	Sum of Factors
−1, −12	−13
−2, −6	−8

The correct factors are −2 and −6.

$x^2 - 8x + 12 = (x + m)(x + n)$ $\quad$ Write the pattern.

$= [x + (-2)][x + (-6)]$ $\quad$ $m = -2$ and $n = -6$

$= (x - 2)(x - 6)$ $\quad$ Simplify.

c. $x^2 + 14x - 15$ $\leftarrow$ b is positive and c is negative.

In this trinomial, $b = 14$ and $c = -15$. This means that $m + n$ is positive and mn is negative. So either m or n must be negative, but not both.

Factors of 12	Sum of Factors
1, −15	−14
−1, 15	14

The correct factors are −1 and 15.

$x^2 + 14x - 15 = (x + m)(x + n)$ $\quad$ Write the pattern.

$= [x + (-1)](x + 15)$ $\quad$ $m = -1$ and $n = 15$

$= (x - 1)(x + 15)$ $\quad$ Simplify.

To factor quadratic trinomials of the form $ax^2 + bx + c$, find two integers m and n whose product is equal to ac and whose sum is equal to b. Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

EXAMPLE

3 **Factor $6x^2 + 7x - 3$.**

In this trinomial, $a = 6$, $b = 7$ and $c = -3$. Find two numbers with a product of $6 \cdot (-3)$ or -18 and a sum of 7.

Factors of -18	Sum of Factors	
$1, -18$	-17	
$-1, 18$	17	
$2, -9$	-7	
$-2, 9$	7	The correct factors are -2 and 9.

$$
\begin{aligned}
6x^2 + 7x - 3 &= 6x^2 + mx + nx - 3 && \text{Write the pattern.}\\
&= 6x^2 + (-2)x + 9x - 3 && m = -2 \text{ and } n = 9\\
&= (6x^2 - 2x) + (9x - 3) && \text{Group terms with common factors.}\\
&= 2x(3x - 1) + 3(3x - 1) && \text{Factor the GCF from each group.}\\
&= (2x + 3)(3x - 1) && \text{Distributive Property}
\end{aligned}
$$

Here are some special products.

Perfect Square Trinomials

$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Difference of Squares

$a^2 - b^2 = (a + b)(a - b)$

EXAMPLE

4 **Factor each polynomial.**

a. $4x^2 + 20x + 25$

The first and last terms are perfect squares.
The middle term is equal to $2(2x)(5)$.
This is a perfect square trinomial of the form $(a + b)^2$.

$$
\begin{aligned}
4x^2 + 20x + 25 &= (2x)^2 + 2(2x)(5) + 5^2 && \text{Write as } a^2 + 2ab + b^2.\\
&= (2x + 5)^2 && \text{Factor using the pattern.}
\end{aligned}
$$

b. $x^2 - 4$

This is a difference of squares.

$$
\begin{aligned}
x^2 - 4 &= x^2 - (2)^2 && \text{Write in the form } a^2 - b^2.\\
&= (x + 2)(x - 2) && \text{Factor the difference of squares.}
\end{aligned}
$$

Exercises

Factor the following polynomials.

1. $12x^2 + 4x$ **2.** $6x^2 y + 2x$ **3.** $8ab^2 - 12ab$

4. $x^2 + 5x + 4$ **5.** $y^2 + 12y + 27$ **6.** $x^2 + 6x + 8$

7. $3y^2 + 13y + 4$ **8.** $7x^2 + 51x + 14$ **9.** $3x^2 + 28x + 32$

10. $x^2 - 5x + 6$ **11.** $y^2 - 5y + 4$ **12.** $6x^2 - 13x + 5$

13. $6a^2 - 50ab + 16b^2$ **14.** $11x^2 - 78x + 7$ **15.** $18x^2 - 31xy + 6y^2$

16. $x^2 + 4xy + 4y^2$ **17.** $9x^2 - 24x + 16$ **18.** $4a^2 + 12ab + 9b^2$

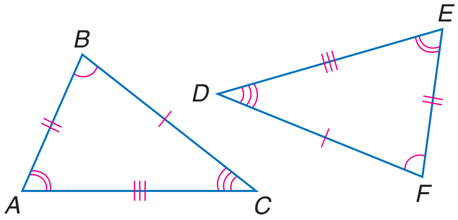
19. $x^2 - 144$ **20.** $4c^2 - 9$ **21.** $16y^2 - 1$

22. $25x^2 - 4y^2$ **23.** $36y^2 - 16$ **24.** $9a^2 - 49b^2$

3 Congruent and Similar Figures

Congruent figures have the same size and the same shape.

Two polygons are congruent if their corresponding sides are congruent and their corresponding angles are congruent.



$\triangle ABC \cong \triangle EFD$

The order of the vertices indicates the corresponding parts.

Congruent Angles	Congruent Sides
$\angle A \cong \angle E$	$\overline{AB} \cong \overline{EF}$
$\angle B \cong \angle F$	$\overline{BC} \cong \overline{FD}$
$\angle C \cong \angle D$	$\overline{AC} \cong \overline{ED}$

Read the symbol $\cong$ as *is congruent to*.

EXAMPLE

1 The corresponding parts of two congruent triangles are marked on the figure. Write a congruence statement for the two triangles.

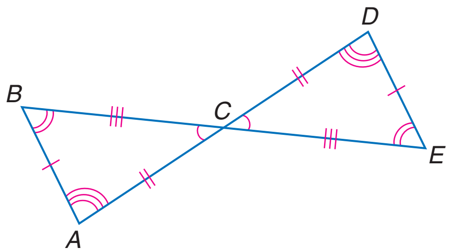
List the congruent angles and sides.

$\angle A \cong \angle D$ $\quad \overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$ $\quad \overline{AC} \cong \overline{DC}$

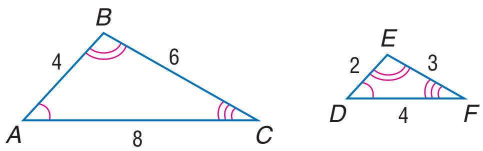
$\angle ACB \cong \angle DCE$ $\quad \overline{BC} \cong \overline{EC}$

Match the vertices of the congruent angles. Therefore, $\triangle ABC \cong \triangle DEC$.



Similar figures have the same shape, but not necessarily the same size.

In similar figures, corresponding angles are congruent, and the measures of corresponding sides are proportional. (They have equivalent ratios.)



Congruent Angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Proportional Sides

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

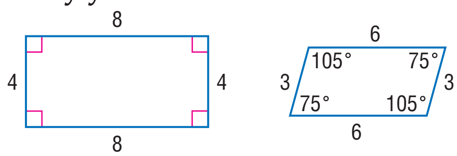
$\triangle ABC \sim \triangle DEF$

Read the symbol $\sim$ as *is similar to.*

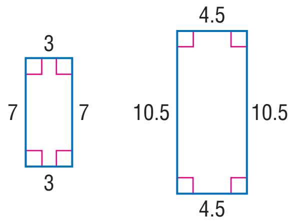
EXAMPLE

2 Determine whether the polygons are similar. Justify your answer.

a. Since $\frac{4}{3} = \frac{8}{6} = \frac{4}{3} = \frac{8}{6}$, the measures of the sides of the polygons are proportional. However, the corresponding angles are not congruent. The polygons are not similar.

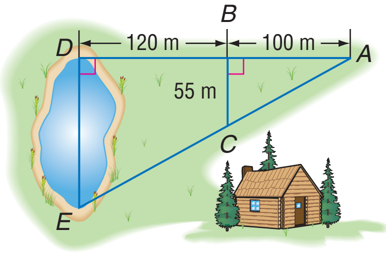


b. Since $\frac{7}{10.5} = \frac{3}{4.5} = \frac{7}{10.5} = \frac{3}{4.5}$, the measures of the sides of the polygons are proportional. The corresponding angles are congruent. Therefore, the polygons are similar.



EXAMPLE

3 **CIVIL ENGINEERING** The city of Mansfield plans to build a bridge across Pine Lake. Use the information in the diagram to find the distance across Pine Lake.



$\triangle ABC \sim \triangle ADE$

$\dfrac{AB}{AD} = \dfrac{BC}{DE}$ — Definition of similar polygons

$\dfrac{100}{220} = \dfrac{55}{DE}$ — $AB = 100$, $AD = 100 + 120 = 220$, $BC = 55$

$100DE = 220(55)$ — Cross products

$100DE = 12{,}100$ — Simplify.

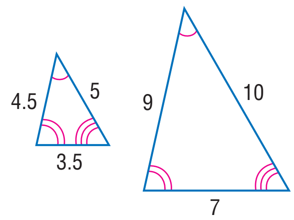
$DE = 121$ — Divide each side by 100.

The distance across the lake is 121 meters.

Exercises

Determine whether each pair of figures is *similar, congruent,* or *neither*.

1.



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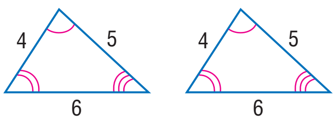

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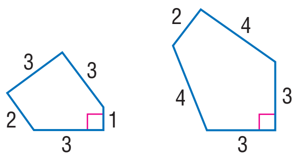

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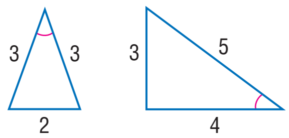

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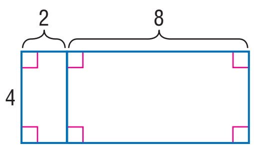

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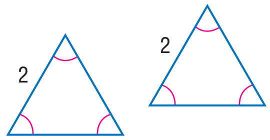
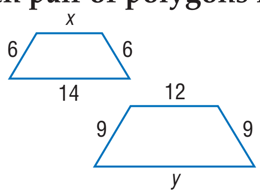


Each pair of polygons is similar. Find the values of x and y.

7.



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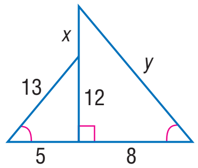

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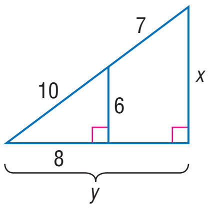
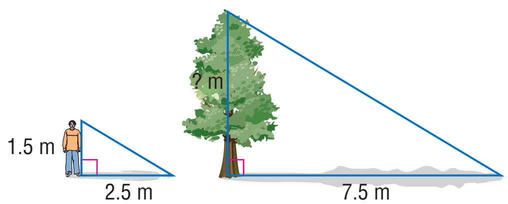


10. **SHADOWS** On a sunny day, Jason measures the length of his shadow and the length of a tree's shadow. Use the figures at the right to find the height of the tree.



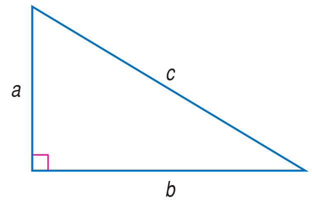
11. **PHOTOGRAPHY** A photo that is 4 inches wide by 6 inches long must be reduced to fit in a space 3 inches wide. How long will the reduced photo be?

12. **SURVEYING** Surveyors use instruments to measure objects that are too large or too far away to measure by hand. They can use the shadows that objects cast to find the height of the objects without measuring them. A surveyor finds that a telephone pole that is 25 feet tall is casting a shadow 20 feet long. A nearby building is casting a shadow 52 feet long. What is the height of the building?

4 Pythagorean Theorem

The **Pythagorean Theorem** states that in a right triangle, the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b.

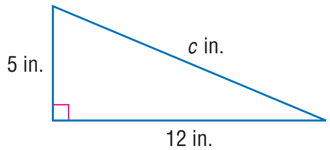
That is, in any right triangle, $c^2 = a^2 + b^2$.



EXAMPLE

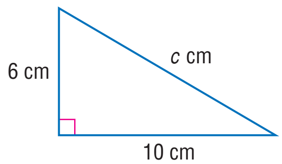
1 Find the length of the hypotenuse of each right triangle.

a.



$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 5^2 + 12^2$	Replace a with 5 and b with 12.
$c^2 = 25 + 144$	Simplify.
$c^2 = 169$	Add.
$c = \sqrt{169}$	Take the square root of each side.
$c = 13$	The length of the hypotenuse is 13 inches.

b.

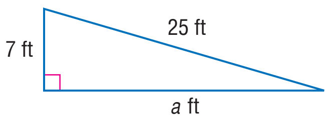


$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 6^2 + 10^2$	Replace a with 6 and b with 10.
$c^2 = 36 + 100$	Simplify.
$c^2 = 136$	Add.
$c = \sqrt{136}$	Take the square root of each side.
$c \approx 11.7$	Use a calculator. To the nearest tenth, the length of the hypotenuse is 11.7 centimeters.

EXAMPLE

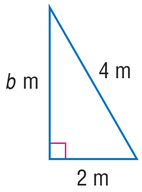
2 Find the length of the missing leg in each right triangle.

a.



$c^2 = a^2 + b^2$	Pythagorean Theorem
$25^2 = a^2 + 7^2$	Replace c with 25 and b with 7.
$625 = a^2 + 49$	Simplify.
$625 - 49 = a^2 + 49 - 49$	Subtract 49 from each side.
$576 = a^2$	Simplify.
$\sqrt{576} = a$	Take the square root of each side.
$24 = a$	The length of the leg is 24 feet.

b.



$c^2 = a^2 + b^2$ Pythagorean Theorem

$4^2 = 2^2 + b^2$ Replace c with 4 and a with 2.

$16 = 4 + b^2$ Simplify.

$12 = b^2$ Subtract 4 from each side.

$\sqrt{12} = b$ Take the square root of each side.

$3.5 \approx b$ Use a calculator to find the square root of 12. Round to the nearest tenth.

To the nearest tenth, the length of the leg is 3.5 meters.

EXAMPLE

3 The lengths of the three sides of a triangle are 5, 7, and 9 inches. Determine whether this triangle is a right triangle.

Since the longest side is 9 inches, use 9 as c, the measure of the hypotenuse.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$9^2 \stackrel{?}{=} 5^2 + 7^2$ Replace c with 9, a with 5, and b with 7.

$81 \stackrel{?}{=} 25 + 49$ Evaluate 9^2, 5^2, and 7^2.

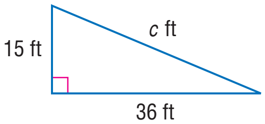
$81 \neq 74$ Simplify.

Since $c^2 \neq a^2 + b^2$, the triangle is *not* a right triangle.

Exercises

Find each missing measure. Round to the nearest tenth, if necessary.

1.



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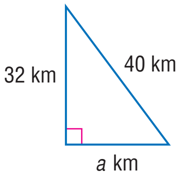
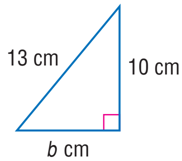


3.



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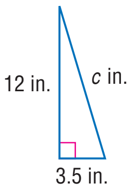

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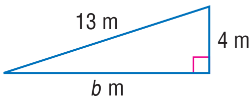

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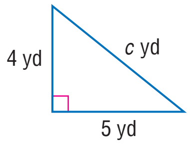


7. $a = 3, b = 4, c = ?$

8. $a = ?, b = 12, c = 13$

9. $a = 14, b = ?, c = 50$

10. $a = 2, b = 9, c = ?$

11. $a = 6, b = ?, c = 13$

12. $a = ?, b = 7, c = 11$

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

13. 5 in., 7 in., 8 in.

14. 9 m, 12 m, 15 m

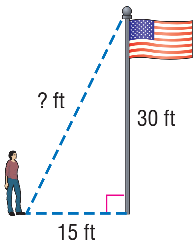
15. 6 cm, 7 cm, 12 cm

16. 11 ft, 12 ft, 16 ft

17. 10 yd, 24 yd, 26 yd

18. 11 km, 60 km, 61 km

19. **FLAGPOLES** Mai-Lin wants to find the distance from her feet to the top of the flagpole. If the flagpole is 30 feet tall and Mai-Lin is standing a distance of 15 feet from the flagpole, what is the distance from her feet to the top of the flagpole?



20. **CONSTRUCTION** The walls of the Downtown Recreation Center are being covered with paneling. The doorway into one room is 0.9 meter wide and 2.5 meters high. What is the width of the widest rectangular panel that can be taken through this doorway?

5 Mean, Median, and Mode

Mean, median, and mode are measures of central tendency that are often used to represent a set of data.

- To find the **mean**, find the sum of the data and divide by the number of items in the data set. (The mean is often called the average.)
- To find the **median**, arrange the data in numerical order. The median is the middle number. If there is an even number of data, the median is the mean of the two middle numbers.
- The **mode** is the number (or numbers) that appears most often in a set of data. If no item appears most often, the set has no mode.

EXAMPLE

1 Michelle is saving to buy a car. She saved \$200 in June, \$300 in July, \$400 in August, and \$150 in September. What was her mean (or average) monthly savings?

mean = sum of monthly savings/number of months

$$= \frac{\$200 + \$300 + \$400 + \$150}{4}$$

$$= \frac{\$1050}{4} \text{ or } \$262.50$$

Michelle's mean monthly savings was \$262.50.

EXAMPLE

2 Find the median of the data.

Peter's Best Running Times	
Week	Minutes to Run a Mile
1	4.5
2	3.7
3	4.1
4	4.1
5	3.6
6	3.4

To find the median, order the numbers from least to greatest. The median is in the middle. The two middle numbers are 3.7 and 4.1.

$$\frac{3.7 + 4.1}{2} = 3.9$$

There is an even number of data. Find the mean of the middle two.

EXAMPLE

3 GOLF Four players tied for first in the 2001 PGA Tour Championship. The scores for each player for each round are shown in the table below. What is the mode score?

Player	Round 1	Round 2	Round 3	Round 4
Mike Weir	68	66	68	68
David Toms	73	66	64	67
Sergio Garcia	69	67	66	68
Ernie Els	69	68	65	68

Source: ESPN

The mode is the score that occurred most often. Since the score of 68 occurred 6 times, it is the mode of these data.

The **range** of a set of data is the difference between the greatest and the least values of the set. It describes how a set of data varies.

EXAMPLE

4 **Find the range of the data.** **{6, 11, 18, 4, 9, 15, 6, 3}**

The greatest value is 18 and the least value is 3. So, the range is 18 − 3 or 15.

Exercises

Find the mean, median, mode, and range for each set of data. Round to the nearest tenth if necessary.

1. {2, 8, 12, 13, 15}
2. {66, 78, 78, 64, 34, 88}
3. {87, 95, 84, 89, 100, 82}
4. {99, 100, 85, 96, 94, 99}
5. {9.9, 9.9, 10, 9.9, 8.8, 9.5, 9.5}
6. {501, 503, 502, 502, 502, 504, 503, 503}
7. {7, 19, 15, 13, 11, 17, 9}
8. {6, 12, 21, 43, 1, 3, 13, 8}
9. {0.8, 0.04, 0.9, 1.1, 0.25}
10. $\left\{2\frac{1}{2}, 1\frac{7}{8}, 2\frac{5}{8}, 2\frac{3}{4}, 2\frac{1}{8}\right\}$

11. **CHARITY** The table shows the amounts collected by classes at Jackson High School. Find the mean, median, mode, and range of the data.

Amounts Collected for Charity			
Class	Amount	Class	Amount
A	$150	E	$10
B	$300	F	$25
C	$55	G	$200
D	$40	H	$100

12. **SCHOOL** The table shows Pilar's grades in chemistry class for the semester. Find her mean, median, and mode scores, and the range of her scores.

Chemistry Grades	
Assignment	Grade (out of 100)
Homework	100
Electron Project	98
Test I	87
Atomic Mass Project	95
Test II	88
Phase Change Project	90
Test III	95

13. **WEATHER** The table shows the precipitation for the month of July in Cape Hatteras, North Carolina, in various years. Find the mean, median, mode, and range of the data.

July Precipitation in Cape Hatteras, North Carolina												
Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Inches	4.22	8.58	5.28	2.03	3.93	1.08	9.54	4.94	10.85	2.66	6.04	3.26

Source: National Climatic Data Center

14. **SCHOOL** Kaitlyn's scores on her first five algebra tests are 88, 90, 91, 89, and 92. What test score must Kaitlyn earn on the sixth test so that her mean score will be at least 90?

15. **GOLF** Colin's average for three rounds of golf is 94. What is the highest score he can receive for the fourth round to have an average (mean) of 92?

16. **SCHOOL** Mika has a mean score of 21 on his first four Spanish quizzes. If each quiz is worth 25 points, what is the highest possible mean score he can have after the fifth quiz?

17. **SCHOOL** To earn a grade of B in math, Latisha must have an average (mean) score of at least 84 on five math tests. Her scores on the first three tests are 85, 89, and 82. What is the lowest total score that Latisha must have on the last two tests to earn a B test average?

6 Bar and Line Graphs

A **bar graph** compares different categories of data by showing each as a bar whose length is related to the frequency. A **double bar graph** compares two sets of data. Another way to represent data is by using a **line graph**. A line graph usually shows how data changes over a period of time.

EXAMPLE

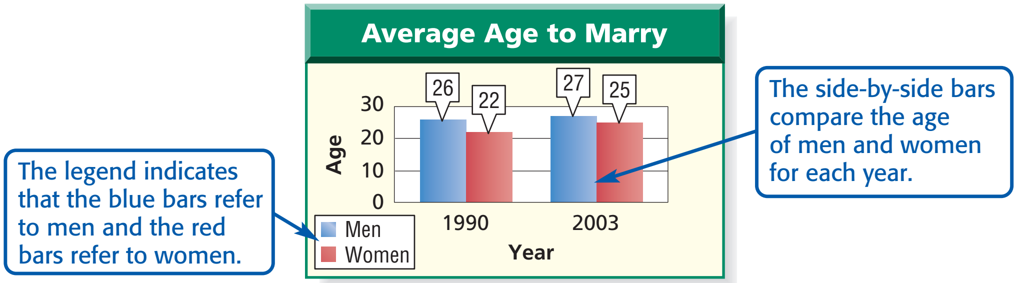
1 **MARRIAGE** **The table shows the average age at which Americans marry for the first time. Make a double bar graph to display the data.**

Average Age to Marry		
Year	**1990**	**2003**
Men	26	27
Women	22	25

Source: U.S. Census Bureau

Step 1 Draw a horizontal and a vertical axis and label them as shown.

Step 2 Draw side-by-side bars to represent each category.



EXAMPLE

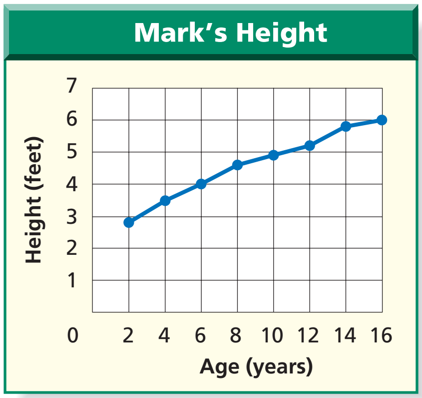
2 **HEALTH** **The table shows Mark's height at 2-year intervals. Make a line graph to display the data.**

Age	2	4	6	8	10	12	14	16
Height (feet)	2.8	3.5	4.0	4.6	4.9	5.2	5.8	6

Step 1 Draw a horizontal and a vertical axis. Label them as shown.

Step 2 Plot the points.

Step 3 Draw a line connecting each pair of consecutive points.



Exercises

1. **HEALTH** The table below shows the life expectancy for Americans born in each year listed. Make a double-bar graph to display the data.

Life Expectancy		
Year of Birth	**Male**	**Female**
1980	70.0	77.5
1985	71.2	78.2
1990	71.8	78.8
1995	72.5	78.9
1998	73.9	79.4

2. **MONEY** The amount of money in Becky's savings account from August through March is shown in the table below. Make a line graph to display the data.

Month	Amount	Month	Amount
August	$300	December	$780
September	$400	January	$800
October	$700	February	$950
November	$780	March	$900

7 Frequency Tables and Histograms

A **frequency table** shows how often an item appears in a set of data. A tally mark is used to record each response. The total number of marks for a given response is the *frequency* of that response. Frequencies can be shown in a bar graph called a histogram. A **histogram** differs from other bar graphs in that no space is between the bars and the bars usually represent numbers grouped by intervals.

EXAMPLE

1 TELEVISION **Use the frequency table of Brad's data.**

Favorite Television Shows		
Program	**Tally**	**Frequency**
Sports	卌 II	7
Mysteries	IIII	4
Soap operas	卌	5
News	卌	5
Quiz shows	卌 I	6
Music videos	II	2
Adventure	卌 IIII	9
Comedies	卌 II	7

a. How many more chose sports programs than news?

b. Which two programs together have the same frequency as adventures?

a. Seven people chose sports. Five people chose news. $7 - 5 = 2$, so 2 more people chose sports than news.

b. As many people chose adventures as the following pairs of programs.

sports and music videos — mysteries and soap operas
mysteries and news — comedies and music videos

EXAMPLE

2 FITNESS **A gym teacher tested the number of sit-ups students in two classes could do in 1 minute. The results are shown.**

Number of Sit-Ups	Frequency
0–4	8
5–9	12
10–14	15
15–19	6
20–24	18
25–29	10

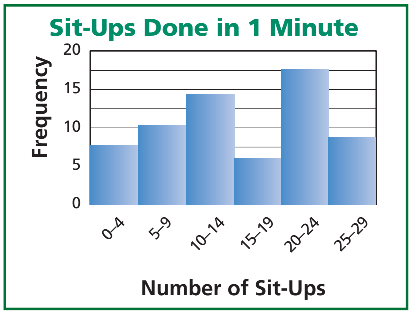
a. Make a histogram of the data. Title the histogram.

b. How many students were able to do 25–29 sit-ups in 1 minute?

c. How many students were unable to do 10 sit-ups in 1 minute?

d. Between which two consecutive intervals does the greatest increase in frequency occur? What is the increase?

a. Use the same intervals as those in the frequency table on the horizontal axis. Label the vertical axis with a scale that includes the frequency numbers from the table.



b. Ten students were able to do 25–29 sit-ups in 1 minute.

c. Add the students who did 0–4 sit-ups and 5–9 sit-ups. So $8 + 12$, or 20, students were unable to do 10 sit-ups in 1 minute.

d. The greatest increase is between intervals 15–19 and 20–24. These frequencies are 6 and 18. So the increase is $18 - 6 = 12$.

Exercises

ART **For Exercises 1–4, use the following information.**

The prices in dollars of paintings sold at an art auction are shown.

1800	**750**	**600**	**600**	**1800**	**1350**	**300**	**1200**	**750**	**600**	**750**	**2700**
600	**750**	**300**	**750**	**600**	**450**	**2700**	**1200**	**600**	**450**	**450**	**300**

1. Make a frequency table of the data.
2. What price was paid most often for the artwork?
3. What is the average price paid for artwork at this auction?
4. How many works of art sold for at least $600 and no more than $1200?

PETS **For Exercises 5–9, use the following information.**

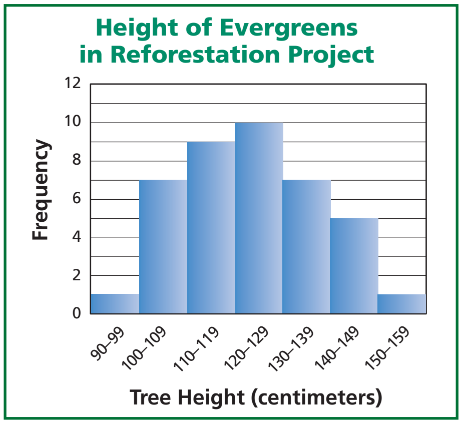
Number of Pets per Family

1	2	3	1	0	2	1	0
1	0	1	4	1	2	0	0
0	1	1	2	2	5	1	0

5. Use a frequency table to make a histogram of the data.
6. How many families own two to three pets?
7. How many families own more than three pets?
8. To the nearest percent, what percent of families own no pets?
9. Name the median, mode, and range of the data.

TREES **For Exercises 10–12, use the histogram shown.**

10. Which interval contains the most evergreen seedlings?
11. Which intervals contain an equal number of trees?
12. Which intervals contain 95% of the data?
13. Between which two consecutive intervals does the greatest increase in frequency occur? What is the increase?



14. **MARKET RESEARCH** A civil engineer is studying traffic patterns. She counts the number of cars that make it through one rush hour green light cycle. Organize her data into a frequency table, and then make a histogram.

    15 16 10 8 8 14 9 7 6 9 10 11 14 10 7 8 9 11 14 10

8 Stem-and-Leaf Plots

In a **stem-and-leaf plot**, data are organized in two columns. The greatest place value of the data is used for the stems. The next greatest place value forms the leaves. Stem-and-leaf plots are useful for organizing long lists of numbers.

EXAMPLE

SCHOOL Isabella has collected data on the GPAs (grade point average) of the 16 students in the art club. Display the data in a stem-and-leaf plot. {4.0, 3.9, 3.1, 3.9, 3.8, 3.7, 1.8, 2.6, 4.0, 3.9, 3.5, 3.3, 2.9, 2.5, 1.1, 3.5}

Step 1 Find the least and the greatest number. Then identify the greatest place-value digit in each number. In this case, ones.

least data: 1.1 — The least number has 1 in the ones place.

greatest data: 4.0 — The greatest number has 4 in the ones place.

Step 2 Draw a vertical line and write the stems from 1 to 4 to the left of the line.

Step 3 Write the leaves to the right of the line, with the corresponding stem. For example, write 0 to the right of 4 for 4.0.

Stem	Leaf
1	8 1
2	5 6 9
3	9 1 9 8 7 9 5 3 5
4	0 0

Step 4 Rearrange the leaves so they are ordered from least to greatest.

Step 5 Include a key or an explanation.

Stem	Leaf
1	1 8
2	5 6 9
3	1 3 5 5 7 8 9 9 9
4	0 0

$3|1 = 3.1$

Exercises

GAMES For Exercises 1–4, use the following information.

The stem-and-leaf plot at the right shows Charmaine's scores for her favorite computer game.

Stem	Leaf
9	0 0 0 1 3 4 5 5 7 8 8 8 9 9
10	0 3 4 4 5 6 9
11	0 3 9 9
12	1 2 6
13	0

$12|6 = 126$

1. What are Charmaine's highest and lowest scores?
2. Which score(s) occurred most frequently?
3. How many scores were above 115?
4. Has Charmaine ever scored 123?
5. **SCHOOL** The class scores on a 50-item test are shown in the table at the right. Make a stem-and-leaf plot of the data.

Test Scores

45	15	30	40	28	35
39	29	38	18	43	49
46	44	48	35	36	30

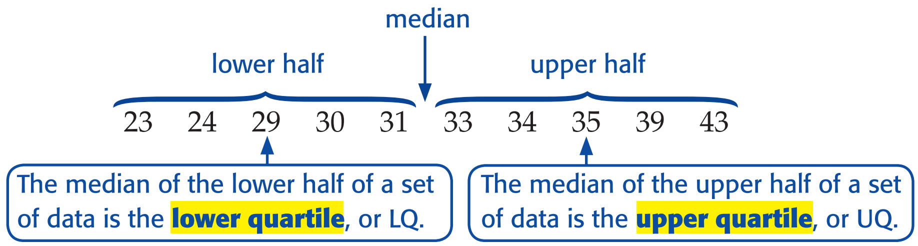
6. **GEOGRAPHY** The table shows the land area of each county in Wyoming. Round each area to the nearest hundred square miles and organize the data in a stem-and-leaf plot.

County	Area (mi)^2	County	Area (mi)^2	County	Area (mi)^2
Albany	4273	Hot Springs	2004	Sheridan	2523
Big Horn	3137	Johnson	4166	Sublette	4883
Campbell	4797	Laramie	2686	Sweetwater	10,425
Carbon	7896	Lincoln	4069	Teton	4008
Converse	4255	Natrona	5340	Unita	2082
Crook	2859	Niobrara	2626	Washakie	2240
Fremont	9182	Park	6942	Weston	2398
Goshen	2225	Platte	2085		

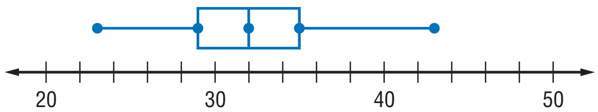
Source: *The World Almanac*

9 Box-and-Whisker Plots

In a set of data, **quartiles** are values that divide the data into four equal parts.



To make a **box-and-whisker plot**, draw a box around the quartile values, and lines or *whiskers* to represent the values in the lower fourth of the data and the upper fourth of the data.

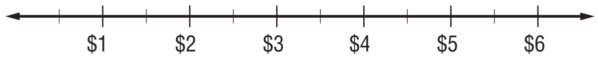


EXAMPLE

1 MONEY The amount spent in the cafeteria by 20 students is shown. Display the data in a box-and-whisker plot.

Amount Spent			
$2.00	$2.00	$1.00	$4.00
$1.00	$2.50	$2.50	$2.00
$2.50	$1.00	$4.00	$2.50
$3.50	$2.00	$3.00	$2.50
$4.00	$4.00	$5.50	$1.50

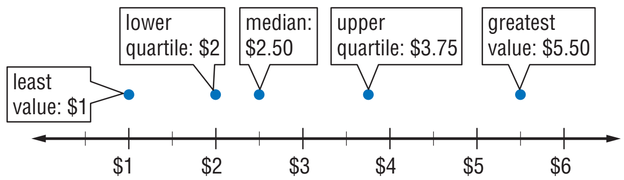
Step 1 Find the least and greatest number. Then draw a number line that covers the range of the data. In this case, the least value is 1 and the greatest value is 5.5.



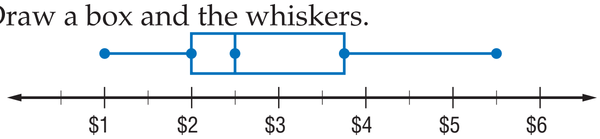
Step 2 Find the median, the extreme values, and the upper and lower quartiles. Mark these points above the number line.

1, 1, 1, 1.5, 2, 2, 2, 2, 2.5, 2.5, 2.5, 2.5, 2.5, 3, 3.5, 4, 4, 4, 4, 5.5

$LQ = \frac{2+2}{2}$ or 2 | $M = \frac{2.5+2.5}{2}$ or 2.5 | $UQ = \frac{3.5+4}{2}$ or 3.75



Step 3 Draw a box and the whiskers.



The **interquartile range (IQR)** is the range of the middle half of the data and contains 50% of the data in the set.

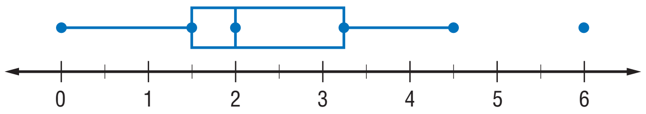
$$\text{Interquartile range} = UQ - LQ$$

The interquartile range of the data in Example 1 is 3.75 − 2 or 1.75.

An **outlier** is any element of a set that is at least 1.5 interquartile ranges less than the lower quartile or greater than the upper quartile. The whisker representing the data is drawn from the box to the least or greatest value that is not an outlier.

EXAMPLE

2 SCHOOL The number of hours José studied each day for the last month is shown in the box-and-whisker plot below.



a. What percent of the data lies between 1.5 and 3.25?
The value 1.5 is the lower quartile and 3.25 is the upper quartile. The values between the lower and upper quartiles represent 50% of the data.

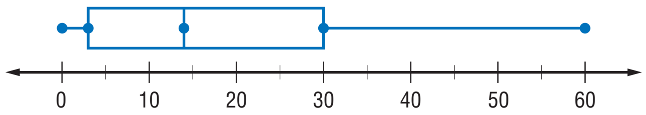
b. What was the greatest amount of time José studied in a day?
The greatest value in the plot is 6, so the greatest amount of time José studied in a day was 6 hours.

c. What is the interquartile range of this box-and-whisker plot?
The interquartile range is UQ − LQ. For this plot, the interquartile range is $3.25 - 1.5$ or 1.75 hours.

d. Identify any outliers in the data.
An outlier is at least 1.5(1.75) less than the lower quartile or more than the upper quartile. Since $3.25 + (1.5)(1.75) = 5.875$, and $6 > 5.875$, the value 6 is an outlier, and was not included in the whisker.

Exercises

DRIVING For Exercises 1–3, use the following information.
Tyler surveyed 20 randomly chosen students at his school about how many miles they drive in an average day. The results are shown in the box-and-whisker plot.



1. What percent of the students drive more than 30 miles in a day?
2. What is the interquartile range of the box-and-whisker plot?
3. Does a student at Tyler's school have a better chance to meet someone who drives the same mileage they do if they drive 50 miles in a day or 15 miles in a day? Why?
4. **SOFT DRINKS** Carlos surveyed his friends to find the number of cans of soft drink they drink in an average week. Make a box-and-whisker plot of the data.
{0, 0, 0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 5, 7, 10, 10, 10, 11, 11}
5. **BASEBALL** The table shows the number of sacrifice hits made by teams in the National Baseball League in one season. Make a box-and-whisker plot of the data.
6. **ANIMALS** The average life span of some animals commonly found in a zoo are as follows: {1, 7, 7, 10, 12, 12, 15, 15, 18, 20, 20, 20, 25, 40, 100}. Make a box-and-whisker plot of the data.

Team	Home Runs	Team	Home Runs
Arizona	71	Milwaukee	65
Atlanta	64	Montreal	64
Chicago	117	New York	52
Cincinnati	66	Philadelphia	67
Colorado	81	Pittsburgh	60
Florida	60	San Diego	29
Houston	71	San Francisco	67
Los Angeles	57	St. Louis	83

Source: ESPN

Extra Practice

Lesson 1-1

(pages 6–10)

Evaluate each expression if $q = \frac{1}{2}$, $r = 1.2$, $s = -6$, and $t = 5$.

1. $qr - st$
2. $qr \div st$
3. $qrst$
4. $qr + st$
5. $\frac{3q}{4s}$
6. $\frac{5qr}{t}$
7. $\frac{2r(4s - 1)}{t}$
8. $\frac{4q^3s + 1}{t - 1}$

Evaluate each expression if $a = -0.5$, $b = 4$, $c = 5$, and $d = -3$.

9. $3b + 4d$
10. $ab^2 + c$
11. $bc + d \div a$
12. $7ab - 3d$
13. $ad + b^2 - c$
14. $\frac{4a + 3c}{3b}$
15. $\frac{3ab^2 - d^3}{a}$
16. $\frac{5a + ad}{bc}$

Lesson 1-2

(pages 11–17)

Name the sets of numbers to which each number belongs. (Use N, W, Z, Q, I, and R.)

1. 8.2
2. -9
3. $\sqrt{36}$
4. $-\frac{1}{3}$
5. $\sqrt{2}$
6. $-0.\overline{24}$

Name the property illustrated by each equation.

7. $(4 + 9a)2b = 2b(4 + 9a)$
8. $3\left(\frac{1}{3}\right) = 1$
9. $a(3 - 2) = a \cdot 3 - a \cdot 2$
10. $(-3b) + 3b = 0$
11. $jk + 0 = jk$
12. $(2a)b = 2(ab)$

Simplify each expression.

13. $7s + 9t + 2s - 7t$
14. $6(2a + 3b) + 5(3a - 4b)$
15. $4(3x - 5y) - 8(2x + y)$
16. $0.2(5m - 8) + 0.3(6 - 2m)$
17. $\frac{1}{2}(7p + 3q) + \frac{3}{4}(6p - 4q)$
18. $\frac{4}{5}(3v - 2w) - \frac{1}{5}(7v - 2w)$

Lesson 1-3

(pages 18–26)

Write an algebraic expression to represent each verbal expression.

1. twelve decreased by the square of a number
2. twice the sum of a number and negative nine
3. the product of the square of a number and 6
4. the square of the sum of a number and 11

Name the property illustrated by each statement.

5. If $a + 1 = 6$, then $3(a + 1) = 3(6)$.
6. If $x + (4 + 5) = 21$, then $x + 9 = 21$.
7. If $7x = 42$, then $7x - 5 = 42 - 5$.
8. If $3 + 5 = 8$ and $8 = 2 \cdot 4$, then $3 + 5 = 2 \cdot 4$.

Solve each equation. Check your solution.

9. $5t + 8 = 88$
10. $27 - x = -4$
11. $\frac{3}{4}y = \frac{2}{3}y + 5$
12. $8s - 3 = 5(2s + 1)$
13. $3(k - 2) = k + 4$
14. $0.5z + 10 = z + 4$
15. $8q - \frac{q}{3} = 46$
16. $-\frac{2}{7}r + \frac{3}{7} = 5$
17. $d - 1 = \frac{1}{2}(d - 2)$

Solve each equation or formula for the specified variable.

18. $C = \pi r$; for r
19. $I = Prt$, for t
20. $m = \frac{n - 2}{n}$, for n

Lesson 1-4

(pages 27–31)

Evaluate each expression if $x = -5$, $y = 3$, and $z = -2.5$.

1. $|2x|$
2. $|-3y|$
3. $|2x + y|$
4. $|y + 5z|$
5. $-|x + z|$
6. $8 - |5y - 3|$
7. $2|x| - 4|2 + y|$
8. $|x + y| - 6|z|$

Solve each equation. Check your solutions.

9. $|d + 1| = 7$
10. $|a - 6| = 10$
11. $2|x - 5| = 22$
12. $|t + 9| - 8 = 5$
13. $|p + 1| + 10 = 5$
14. $6|g - 3| = 42$
15. $2|y + 4| = 14$
16. $|3b - 10| = 2b$
17. $|3x + 7| + 4 = 0$
18. $|2c + 3| - 15 = 0$
19. $7 - |m - 1| = 3$
20. $3 + |z + 5| = 10$
21. $2|2d - 7| + 1 = 35$
22. $|3t + 6| + 9 = 30$
23. $|d - 3| = 2d + 9$
24. $|4y - 5| + 4 = 7y + 8$
25. $|2b + 4| - 3 = 6b + 1$
26. $|5t| + 2 = 3t + 18$

Lesson 1-5

(pages 33–39)

Solve each inequality. Then graph the solution set on a number line.

1. $2z + 5 \leq 7$
2. $3r - 8 > 7$
3. $0.75b < 3$
4. $-3x > 6$
5. $2(3f + 5) \geq 28$
6. $-33 > 5g + 7$
7. $-3(y - 2) \geq -9$
8. $7a + 5 > 4a - 7$
9. $5(b - 3) \leq b - 7$
10. $3(2x - 5) < 5(x - 4)$
11. $8(2c - 1) > 11c + 22$
12. $2(d + 4) - 5 \geq 5(d + 3)$
13. $8 - 3t < 4(3 - t)$
14. $-x \geq \frac{x + 4}{7}$
15. $\frac{a + 8}{4} \leq \frac{7 + a}{3}$
16. $-y < \frac{y + 5}{2}$
17. $5(x - 1) - 4x \geq 3(3 - x)$
18. $6s - (4s + 7) > 5 - s$

Define a variable and write an inequality for each problem. Then solve.

19. The product of 7 and a number is greater than 42.
20. The difference of twice a number and 3 is at most 11.
21. The product of -10 and a number is greater than or equal to 20.
22. Thirty increased by a number is less than twice the number plus three.

Lesson 1-6

(pages 41–48)

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

1. all numbers less than -9 and greater than 9
2. all numbers between -5.5 and 5.5
3. all numbers greater than or equal to -2 and less than or equal to 2

Solve each inequality. Graph the solution set on a number line.

4. $3m - 2 < 7$ or $2m + 1 > 13$
5. $2 < n + 4 < 7$
6. $-3 \leq s - 2 \leq 5$
7. $5t + 3 \leq -7$ or $5t - 2 \geq 8$
8. $7 \leq 4x + 3 \leq 19$
9. $4x + 7 < 5$ or $2x - 4 > 12$
10. $|7x| \geq 21$
11. $|8p| \leq 16$
12. $|7d| \geq -42$
13. $|a + 3| < 1$
14. $|t - 4| > 1$
15. $|2y - 5| < 3$
16. $|3d + 6| \geq 3$
17. $|4x - 1| < 5$
18. $|6v + 12| > 18$
19. $|2r + 4| < 6$
20. $|5w - 3| \geq 9$
21. $|z + 2| \geq 0$
22. $12 + |2q| < 0$
23. $|3h| + 15 < 0$
24. $|5n - 16| \geq 4$

Lesson 2-1

(pages 58–64)

State the domain and range of each relation. Then determine whether each relation is a function. Write *yes* or *no*.

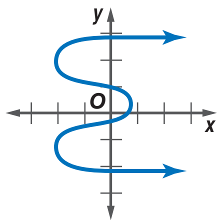
1.

Year	Population
1970	11,605
1980	13,468
1990	15,630
2000	18,140

2.

x	y
1	5
2	5
3	5
4	5

3.



Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function and state whether *discrete* or *continuous*.

4. $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$
5. $\{(0, 3), (0, 2), (0, 1), (0, 0)\}$
6. $y = -x$
7. $y = 2x - 1$
8. $y = 2x^2$
9. $y = -x^2$

Find each value if $f(x) = x + 7$ and $g(x) = (x + 1)^2$.

10. $f(2)$
11. $f(-4)$
12. $f(a + 2)$
13. $g(4)$
14. $g(-2)$
15. $f(0.5)$
16. $g(b - 1)$
17. $g(3c)$

Lesson 2-2

(pages 66–70)

State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.

1. $\frac{x}{2} - y = 7$
2. $\sqrt{x} = y + 5$
3. $g(x) = \frac{2}{x - 3}$
4. $f(x) = 7$

Write each equation in standard form. Identify *A*, *B*, and *C*.

5. $x + 7 = y$
6. $x = -3y$
7. $5x = 7y + 3$
8. $y = \frac{2}{3}x + 8$
9. $-0.4x = 10$
10. $0.75y = -6$

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

11. $2x + y = 6$
12. $3x - 2y = -12$
13. $y = -x$
14. $x = 3y$
15. $\frac{3}{4}y - x = 1$
16. $y = -3$

Lesson 2-3

(pages 71–77)

Find the slope of the line that passes through each pair of points.

1. $(0, 3), (5, 0)$
2. $(2, 3), (5, 7)$
3. $(2, 8), (2, -8)$
4. $(1.5, -1), (3, 1.5)$
5. $\left(-\frac{1}{2}, \frac{3}{5}\right), \left(\frac{3}{10}, -\frac{1}{4}\right)$
6. $(-3, c), (4, c)$

Graph the line passing through the given point with the given slope.

7. $(0, 3); 1$
8. $(2, 3); 0$
9. $(-1, 1); -\frac{1}{3}$

Graph the line that satisfies each set of conditions.

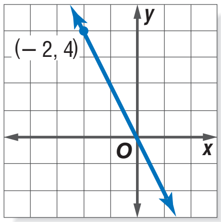
10. passes through $(0, 1)$, parallel to a line with a slope of -2
11. passes through $(4, -5)$, perpendicular to the graph of $-2x + 5y = 1$

Lesson 2-4

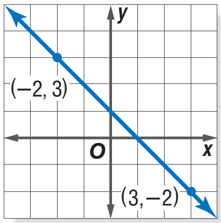
(pages 79–84)

Write an equation in slope-intercept form for each graph.

1.



2.



Write an equation in slope-intercept form for the line that satisfies each set of conditions.

3. slope -1, passes through $(7, 2)$

4. slope $\frac{3}{4}$, passes through the origin

5. passes through $(1, -3)$ and $(-1, 2)$

6. x-intercept -5, y-intercept 2

7. passes through $(1, 1)$, parallel to the graph of $2x + 3y = 5$

8. passes through $(0, 0)$, perpendicular to the graph of $2y + 3x = 4$

Lesson 2-5

(pages 86–91)

Complete parts a–c for each set of data in Exercises 1–3.

a. **Draw a scatter plot and describe the correlation.**

b. **Use two ordered pairs to write a prediction equation.**

c. **Use your prediction equation to predict the missing value.**

1.

Telephone Costs	
Minutes	Cost ($)
1	0.20
3	0.52
4	0.68
6	1.00
9	1.48
15	?

2.

Washington	
Year	Population
1960	2,853,214
1970	3,413,244
1980	4,132,353
1990	4,866,669
2000	5,894,121
2010	?

Source: *The World Almanac*

3.

Federal Minimum Wage	
Year	Wage
1981	$3.35
1990	$3.80
1991	$4.25
1996	$4.75
1997	$5.15
2015	?

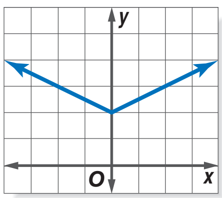
Source: *The World Almanac*

Lesson 2-6

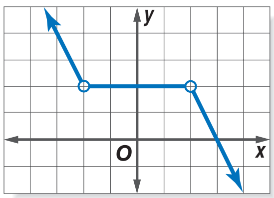
(pages 95–101)

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

1.



2.



Graph each function. Identify the domain and range.

3. $f(x) = [\![x + 5]\!]$

4. $g(x) = [\![x]\!] - 2$

5. $f(x) = -2[\![x]\!]$

6. $h(x) = |x| - 3$

7. $h(x) = |x - 1|$

8. $g(x) = |2x| + 2$

9. $h(x) = \begin{cases} x \text{ if } x < -2 \\ 4 \text{ if } x \geq -2 \end{cases}$

10. $f(x) = \begin{cases} -3 \text{ if } x \leq 1 \\ -x \text{ if } x > 1 \end{cases}$

Lesson 2-7

(pages 102–105)

Graph each inequality.

1. $y \geq x - 2$
2. $y < -3x - 1$
3. $4y \leq -3x + 8$
4. $3x > y$
5. $x + 2 \geq y - 7$
6. $2x < 5 - y$
7. $y > \frac{1}{5}x - 8$
8. $2y - 5x \leq 8$
9. $-2x + 5 \leq \frac{2}{3}y$
10. $3x + 2y \geq 0$
11. $x \leq 2$
12. $\frac{y}{2} \leq x - 1$
13. $y - 3 < 5$
14. $y \geq -|x|$
15. $|x| \leq y + 3$
16. $y > |5x - 3|$
17. $y \leq |8 - x|$
18. $y < |x + 3| - 1$
19. $y + |2x| \geq 4$
20. $y \geq |2x - 1| + 5$
21. $y < \left|\frac{2x}{3}\right| - 1$

Lesson 3-1

(pages 116–122)

Solve each system of equations by graphing or by completing a table.

1. $x + 3y = 18$, $-x + 2y = 7$
2. $x - y = 2$, $2x - 2y = 10$
3. $2x + 6y = 6$, $\frac{1}{3}x + y = 1$
4. $x + 3y = 0$, $2x + 6y = 5$
5. $2x - y = 7$, $\frac{2}{5}x - \frac{4}{3}y = -2$
6. $y = \frac{1}{3}x + 1$, $y = 4x + 1$

Graph each system of equations and describe it as ***consistent and independent, consistent and dependent,*** **or** ***inconsistent.***

7. $2x + 3y = 5$, $-6x - 9y = -15$
8. $x - 2y = 4$, $y = x - 2$
9. $y = 0.5x$, $2y = x + 4$
10. $9x - 5 = 7y$, $4.5x - 3.5y = 2.5$
11. $\frac{3}{4}x - y = 0$, $\frac{1}{3}y + \frac{1}{2}x = 6$
12. $\frac{2}{3}x = \frac{5}{3}y$, $2x - 5y = 0$

Lesson 3-2

(pages 123–129)

Solve each system of equations by using substitution.

1. $2x + 3y = 10$, $x + 6y = 32$
2. $x = 4y - 10$, $5x + 3y = -4$
3. $3x - 4y = -27$, $2x + y = -7$

Solve each system of equations by using elimination.

4. $7x + y = 9$, $5x - y = 15$
5. $r + 5s = -17$, $2r - 6s = -2$
6. $6p + 8q = 20$, $5p - 4q = -26$

Solve each system of equations by using either substitution or elimination.

7. $2x - 3y = 7$, $3x + 6y = 42$
8. $2a + 5b = -13$, $3a - 4b = 38$
9. $3c + 4d = -1$, $6c - 2d = 3$
10. $7x - y = 35$, $y = 5x - 19$
11. $3m + 4n = 28$, $5m - 3n = -21$
12. $x = 2y - 1$, $4x - 3y = 21$
13. $2.5x + 1.5y = -2$, $3.5x - 0.5y = 18$
14. $\frac{5}{2}x + \frac{1}{3}y = 13$, $\frac{1}{2}x - y = -7$
15. $\frac{2}{7}c - \frac{4}{3}d = 16$, $\frac{4}{7}c + \frac{8}{3}d = -16$

Extra Practice

Lesson 3-3
(pages 130–135)

Solve each system of inequalities.

1. $x \le 5$
   $y \ge -3$
2. $y < 3$
   $y - x \ge -1$
3. $x + y < 5$
   $x < 2$
4. $y + x < 2$
   $y \ge x$
5. $x + y \le 2$
   $y - x \le 4$
6. $y \le x + 4$
   $y - x \ge 1$
7. $y < \frac{1}{3}x + 5$
   $y > 2x + 1$
8. $y + x \ge 1$
   $y - x \ge -1$
9. $|x| > 2$
   $|y| \le 5$
10. $|x - 3| \le 3$
    $4y - 2x \le 6$
11. $4x + 3y \ge 12$
    $2y - x \ge -1$
12. $y \le -1$
    $3x - 2y \ge 6$

Find the coordinates of the vertices of the figure formed by each system of inequalities.

13. $y \le 3$
    $x \le 2$
    $y \ge -\frac{3}{2}x + 3$
14. $y \ge -1$
    $y \le x$
    $y \le -x + 4$
15. $y \le \frac{1}{3}x + \frac{7}{3}$
    $4x - y \le 5$
    $y \ge -\frac{3}{2}x + \frac{1}{2}$

Lesson 3-4
(pages 138–144)

A feasible region has vertices at (−3, 2), (1, 3), (6, 1), and (2, −2). Find the maximum and minimum values of each function.

1. $f(x, y) = 2x - y$
2. $f(x, y) = x + 5y$
3. $f(x, y) = y - 4x$
4. $f(x, y) = -x + 3y$
5. $f(x, y) = 3x - y$
6. $f(x, y) = 2y - 2x$

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

7. $4x - 5y \le -10$
   $y \le 6$
   $2x + y \ge 2$
   $f(x, y) = x + y$
8. $x \le 5$
   $y \ge 2$
   $2x - 5y \ge -10$
   $f(x, y) = 3x + y$
9. $x - 2y \ge -7$
   $x + y \le 8$
   $y \ge 5x + 8$
   $f(x, y) = 3x - 4y$
10. $y \le 4x + 6$
    $x + 4y \ge 7$
    $2x + y \le 7$
    $f(x, y) = 2x - y$
11. $y \ge 0$
    $y \le 5$
    $y \le -x + 7$
    $5x + 3y \ge 20$
    $f(x, y) = x + 2y$
12. $y \ge 0$
    $3x - 2y \ge 0$
    $x + 3y \le 11$
    $2x + 3y \le 16$
    $f(x, y) = 4x + y$

Lesson 3-5
(pages 145–152)

For each system of equations, an ordered triple is given. Determine whether or not it is a solution of the system.

1. $4x + 2y - 6z = -38$
   $5x - 4y + z = -18$
   $x + 3y + 7z = 38$;
   $(-3, 2, 5)$
2. $u + 3v + w = 14$
   $2u - v + 3w = -9$
   $4u - 5v - 2w = -2$;
   $(1, 5, -2)$
3. $x + y = -6$
   $x + z = -2$
   $y + z = 2$;
   $(-4, -2, 2)$

Solve each system of equations.

4. $5a = 5$
   $6b - 3c = 15$
   $2a + 7c = -5$
5. $s + 2t = 5$
   $7r - 3s + t = 20$
   $2t = 8$
6. $2u - 3v = 13$
   $3v + w = -3$
   $4u - w = 2$
7. $4a + 2b - c = 5$
   $2a + b - 5c = -11$
   $a - 2b + 3c = 6$
8. $x + 2y - z = 1$
   $x + 3y + 2z = 7$
   $2x + 6y + z = 8$
9. $2x + y - z = 7$
   $3x - y + 2z = 15$
   $x - 4y + z = 2$

Lesson 4-1

(pages 162–167)

Solve each matrix equation.

1. $[2x \quad 3y \quad -z] = [2y \quad -z \quad 15]$

2. $\begin{bmatrix} x+y \\ 4x-3y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$

3. $-2\begin{bmatrix} w+5 & x-z \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 6 & 2x+8z \end{bmatrix}$

4. $y\begin{bmatrix} 2 & x \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -10 \\ 10 & 2z \end{bmatrix}$

5. $\begin{bmatrix} 2x \\ -y \\ 3z \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \\ -21 \end{bmatrix}$

6. $\begin{bmatrix} x-3y \\ 4y-3x \end{bmatrix} = -5\begin{bmatrix} 2 \\ x \end{bmatrix}$

7. $\begin{bmatrix} x^2+4 & y+6 \\ x-y & 2-y \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 0 & 1 \end{bmatrix}$

8. $\begin{bmatrix} x+y & 3 \\ y & 6 \end{bmatrix} = \begin{bmatrix} 0 & 2y-x \\ z & 4-2x \end{bmatrix}$

Lesson 4-2

(pages 169–176)

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 3 & 5 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 8 & -1 \end{bmatrix}$

2. $[0 \quad -1 \quad 3] + \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$

3. $\begin{bmatrix} 45 & 36 & 18 \\ 63 & 29 & 5 \end{bmatrix} - \begin{bmatrix} 45 & -2 & 36 \\ 18 & 9 & -10 \end{bmatrix}$

4. $4[-8 \quad 2 \quad 9] - 3[2 \quad -7 \quad 6]$

5. $5\begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix} - 2\begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix} + 4\begin{bmatrix} 7 & -6 \\ -4 & 2 \end{bmatrix}$

6. $1.3\begin{bmatrix} 3.7 \\ -5.4 \end{bmatrix} + 4.1\begin{bmatrix} 6.4 \\ -3.7 \end{bmatrix} - 6.2\begin{bmatrix} -0.8 \\ 7.4 \end{bmatrix}$

Use matrices A, B, C, D, and E to find the following.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ -3 & 3 \end{bmatrix}$, $D = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$, $E = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$

7. $A + B$
8. $C + D$
9. $A - B$
10. $4B$
11. $D - C$
12. $E + 2A$
13. $D - 2B$
14. $2A + 3E - D$

Lesson 4-3

(pages 177–184)

Find each product, if possible.

1. $[-3 \quad 4] \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

2. $\begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix}$

4. $\begin{bmatrix} 3 & 2 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} -8 \\ 15 \end{bmatrix}$

5. $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 6 & 1 \\ 2 & -4 & 0 \end{bmatrix}$

6. $\begin{bmatrix} 0 & 1 & -2 \\ 5 & 3 & -4 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & -2 \end{bmatrix}$

7. $\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

8. $\begin{bmatrix} -1 & 0 & 2 \\ -6 & 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$

Extra Practice

Lesson 4-4

(pages 185–192)

For Exercises 1–3, use the following information.

The vertices of quadrilateral $ABCD$ are $A(1, 1)$, $B(-2, 3)$, $C(-4, -1)$, and $D(2, -3)$. The quadrilateral is dilated so that its perimeter is 2 times the original perimeter.

1. Write the coordinates for $ABCD$ in a vertex matrix.
2. Find the coordinates of the image $A'B'C'D'$.
3. Graph $ABCD$ and $A'B'C'D'$.

For Exercises 4–10, use the following information.

The vertices of $\triangle MQN$ are $M(2, 4)$, $Q(3, -5)$, and $N(1, -1)$.

4. Write the coordinates of $\triangle MQN$ in a vertex matrix.
5. Write the reflection matrix for reflecting over the line $y = x$.
6. Find the coordinates of $\triangle M'Q'N'$ after the reflection.
7. Graph $\triangle MQN$ and $\triangle M'Q'N'$.
8. Write a rotation matrix for rotating $\triangle MQN$ 90° counterclockwise about the origin.
9. Find the coordinates of $\triangle M'Q'N'$ after the rotation.
10. Graph $\triangle MQN$ and $\triangle M'Q'N'$.

Lesson 4-5

(pages 194–200)

Evaluate each determinant using expansion by minors.

1. $\begin{vmatrix} 2 & -3 & 5 \\ 1 & -2 & -7 \\ -1 & 4 & -3 \end{vmatrix}$
2. $\begin{vmatrix} 0 & -1 & 2 \\ -2 & 1 & 0 \\ 2 & 0 & -1 \end{vmatrix}$
3. $\begin{vmatrix} 4 & 3 & -2 \\ 2 & 5 & -8 \\ 6 & 4 & -1 \end{vmatrix}$
4. $\begin{vmatrix} -3 & 0 & 2 \\ 1 & -2 & -1 \\ 0 & 5 & 0 \end{vmatrix}$

Evaluate each determinant using diagonals.

5. $\begin{vmatrix} 3 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 0 & 3 \end{vmatrix}$
6. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$
7. $\begin{vmatrix} 6 & 4 & -1 \\ 2 & 5 & -8 \\ 4 & 3 & -2 \end{vmatrix}$
8. $\begin{vmatrix} 6 & 12 & 15 \\ 9 & 3 & 14 \\ 5 & 6 & 3 \end{vmatrix}$

Lesson 4-6

(pages 201–207)

Use Cramer's Rule to solve each system of equations.

1. $5x - 3y = 19$
   $7x + 2y = 8$
2. $4p - 3q = 22$
   $2p + 8q = 30$
3. $-x + y = 5$
   $2x + 4y = 38$
4. $\frac{1}{3}x - \frac{1}{2}y = -8$
   $\frac{3}{5}x + \frac{5}{6}y = -4$
5. $\frac{1}{4}c + \frac{2}{3}d = 6$
   $\frac{3}{4}c - \frac{5}{3}d = -4$
6. $0.3a + 1.6b = 0.44$
   $0.4a + 2.5b = 0.66$
7. $x + y + z = 6$
   $2x - y - z = -3$
   $3x + y - 2z = -1$
8. $2a + b - c = -6$
   $a - 2b + c = 8$
   $-a - 3b + 2c = 14$
9. $r + 2s - t = 10$
   $-2r + 3s + t = 6$
   $3r - 2s + 2t = -19$

Lesson 4-7

(pages 208–215)

Determine whether each pair of matrices are inverses.

1. $A = \begin{bmatrix} -7 & -6 \\ 8 & 7 \end{bmatrix}, B = \begin{bmatrix} -7 & -6 \\ 8 & 7 \end{bmatrix}$

2. $C = \begin{bmatrix} -3 & 4 \\ 2 & -2 \end{bmatrix}, D = \begin{bmatrix} -2 & -2 \\ -4 & -3 \end{bmatrix}$

3. $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, M = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 3 & 2 \\ 0 & -4 \end{bmatrix}$

7. $\begin{bmatrix} 3 & 8 \\ 0 & -1 \end{bmatrix}$

8. $\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}$

10. $\begin{bmatrix} 8 & -5 \\ -6 & 4 \end{bmatrix}$

11. $\begin{bmatrix} 10 & 3 \\ 5 & -2 \end{bmatrix}$

12. $\begin{bmatrix} -3 & 4 \\ -4 & 8 \end{bmatrix}$

Lesson 4-8

(pages 216–222)

Write a matrix equation for each system of equations.

1. $5a + 3b = 6$
$2a - b = 9$

2. $3x + 4y = -8$
$2x - 3y = 6$

3. $m + 3n = 1$
$4m - n = -22$

4. $4c - 3d = -1$
$5c - 2d = 39$

5. $x + 2y - z = 6$
$-2x + 3y + z = 1$
$x + y + 3z = 8$

6. $2a - 3b - c = 4$
$4a + b + c = 15$
$a - b - c = -2$

Solve each matrix equation or system of equations.

7. $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 33 \\ -1 \end{bmatrix}$

8. $\begin{bmatrix} -1 & 1 \\ 7 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -29 \\ 52 \end{bmatrix}$

10. $5x - y = 7$
$8x + 2y = 4$

11. $3m + n = 4$
$2m + 2n = 3$

12. $6c + 5d = 7$
$3c - 10d = -4$

13. $3a - 5b = 1$
$a + 3b = 5$

14. $2r - 7s = 24$
$-r + 8s = -21$

15. $x + y = -3$
$3x - 10y = 43$

16. $2m - 3n = 3$
$-4m + 9n = -8$

17. $x + y = 1$
$2x - 2y = -12$

Lesson 5-1

(pages 236–244)

For Exercises 1–12, complete parts a–c for each quadratic function.

a. **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.**

b. **Make a table of values that includes the vertex.**

c. **Use this information to graph the function.**

1. $f(x) = 6x^2$

2. $f(x) = -x^2$

3. $f(x) = x^2 + 5$

4. $f(x) = -x^2 - 2$

5. $f(x) = 2x^2 + 1$

6. $f(x) = -3x^2 + 6x$

7. $f(x) = x^2 + 6x - 3$

8. $f(x) = x^2 - 2x - 8$

9. $f(x) = -3x^2 - 6x + 12$

10. $f(x) = x^2 + 5x - 6$

11. $f(x) = 2x^2 + 7x - 4$

12. $f(x) = -5x^2 + 10x + 1$

Determine whether each function has a maximum or a minimum value and find the maximum or minimum value. Then state the domain and range of the function.

13. $f(x) = 9x^2$

14. $f(x) = 9 - x^2$

15. $f(x) = x^2 - 5x + 6$

16. $f(x) = 2 + 7x - 6x^2$

17. $f(x) = 4x^2 - 9$

18. $f(x) = x^2 + 2x + 1$

19. $f(x) = 8 - 3x - 4x^2$

20. $f(x) = x^2 - x + \frac{5}{4}$

21. $f(x) = -x^2 + \frac{14}{3}x + \frac{5}{3}$

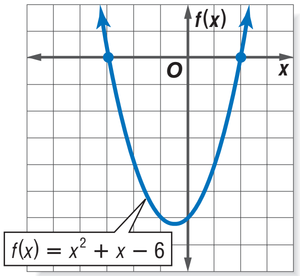
Extra Practice

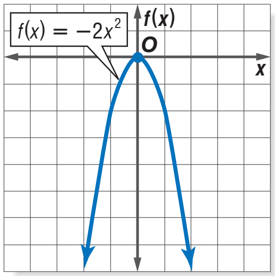
Lesson 5-2

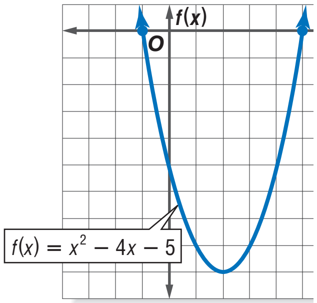
(pages 246–251)

Use the related graph of each equation to determine its solutions.

1. $x^2 + x - 6 = 0$
2. $-2x^2 = 0$
3. $x^2 - 4x - 5 = 0$







Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $x^2 - 2x = 0$
5. $x^2 + 8x - 20 = 0$
6. $-2x^2 + 10x - 5 = 0$
7. $-5x + 2x^2 - 3 = 0$
8. $3x^2 - x + 8 = 0$
9. $-x^2 + 2 = 7x$
10. $4x^2 - 4x + 1 = 0$
11. $4x + 1 = 3x^2$
12. $x^2 = -9x$
13. $x^2 + 6x - 27 = 0$
14. $0.4x^2 + 1 = 0$
15. $0.5x^2 + 3x - 2 = 0$

Lesson 5-3

(pages 253–258)

Solve each equation by factoring.

1. $x^2 + 7x + 10 = 0$
2. $3x^2 = 75x$
3. $2x^2 + 7x = 9$
4. $8x^2 = 48 - 40x$
5. $5x^2 = 20x$
6. $16x^2 - 64 = 0$
7. $24x^2 - 15 = 2x$
8. $x^2 = 72 - x$
9. $4x^2 + 9 = 12x$
10. $2x^2 - 8x = 0$
11. $8x^2 + 10x = 3$
12. $12x^2 - 5x = 3$
13. $x^2 + 9x + 14 = 0$
14. $9x^2 + 1 = 6x$
15. $6x^2 + 7x = 3$
16. $x^2 - 4x = 21$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

17. $2, 1$
18. $-3, 4$
19. $-1, -7$
20. $-1, \frac{1}{2}$
21. $-5, \frac{1}{4}$
22. $-\frac{1}{3}, -\frac{1}{2}$

Lesson 5-4

(pages 259–266)

Simplify.

1. $\sqrt{-289}$
2. $\sqrt{-\frac{25}{121}}$
3. $\sqrt{-625b^8}$
4. $\sqrt{-\frac{28t^6}{27s^5}}$
5. $(7i)^2$
6. $(6i)(-2i)(11i)$
7. $(\sqrt{-8})(\sqrt{-12})$
8. $-i^{22}$
9. $i^{17} \cdot i^{12} \cdot i^{26}$
10. $(14 - 5i) + (-8 + 19i)$
11. $(7i) - (2 + 3i)$
12. $(2 + 2i) - (5 + i)$
13. $(7 + 3i)(7 - 3i)$
14. $(8 - 2i)(5 + i)$
15. $(6 + 8i)^2$
16. $\frac{3}{6 - 2i}$
17. $\frac{5i}{3 + 4i}$
18. $\frac{3 - 7i}{5 + 4i}$

Solve each equation.

19. $x^2 + 8 = 3$
20. $\frac{4x^2}{49} + 6 = 3$
21. $8x^2 + 5 = 1$
22. $12 - 9x^2 = 38$
23. $9x^2 + 7 = 4$
24. $\frac{1}{2}x^2 + 1 = 0$

Lesson 5-5

(pages 268–275)

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

1. $x^2 - 4x + c$
2. $x^2 + 20x + c$
3. $x^2 - 11x + c$
4. $x^2 - \frac{2}{3}x + c$
5. $x^2 + 30x + c$
6. $x^2 + \frac{3}{8}x + c$
7. $x^2 - \frac{2}{5}x + c$
8. $x^2 - 3x + c$

Solve each equation by completing the square.

9. $x^2 + 3x - 4 = 0$
10. $x^2 + 5x = 0$
11. $x^2 + 2x - 63 = 0$
12. $3x^2 - 16x - 35 = 0$
13. $x^2 + 7x + 13 = 0$
14. $5x^2 - 8x + 2 = 0$
15. $x^2 - 6x + 11 = 0$
16. $x^2 - 12x + 36 = 0$
17. $8x^2 + 13x - 4 = 0$
18. $3x^2 + 5x + 6 = 0$
19. $x^2 + 14x - 1 = 0$
20. $4x^2 - 32x + 15 = 0$
21. $3x^2 - 11x - 4 = 0$
22. $x^2 + 8x - 84 = 0$
23. $x^2 - 7x + 5 = 0$
24. $x^2 + 3x - 8 = 0$
25. $x^2 - 5x - 10 = 0$
26. $3x^2 - 12x + 4 = 0$
27. $x^2 + 20x + 75 = 0$
28. $x^2 - 5x - 24 = 0$
29. $2x^2 + x - 21 = 0$

Lesson 5-6

(pages 276–283)

For Exercises 1–16, complete parts a–c for each quadratic equation.

a. **Find the value of the discriminant.**

b. **Describe the number and type of roots.**

c. **Find the exact solutions by using the Quadratic Formula.**

1. $x^2 + 7x + 13 = 0$
2. $6x^2 + 6x - 21 = 0$
3. $5x^2 - 5x + 4 = 0$
4. $9x^2 + 42x + 49 = 0$
5. $4x^2 - 16x + 3 = 0$
6. $2x^2 = 5x + 3$
7. $x^2 + 81 = 18x$
8. $3x^2 - 30x + 75 = 0$
9. $24x^2 + 10x = 43$
10. $9x^2 + 4 = 2x$
11. $7x = 8x^2$
12. $18x^2 = 9x + 45$
13. $x^2 - 4x + 4 = 0$
14. $4x^2 + 16x + 15 = 0$
15. $x^2 - 6x + 13 = 0$

Solve each equation by using the method of your choice. Find exact solutions.

16. $x^2 + 4x + 29 = 0$
17. $4x^2 + 3x - 2 = 0$
18. $2x^2 + 5x = 9$
19. $x^2 = 8x - 16$
20. $7x^2 = 4x$
21. $2x^2 + 6x + 5 = 0$
22. $9x^2 - 30x + 25 = 0$
23. $3x^2 - 4x + 2 = 0$
24. $3x^2 = 108x$

Lesson 5-7

(pages 286–292)

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1. $y = (x + 6)^2 - 1$
2. $y = 2(x - 8)^2 - 5$
3. $y = -(x + 1)^2 + 7$
4. $y = -9(x - 7)^2 + 3$
5. $y = -x^2 + 10x - 3$
6. $y = -2x^2 + 16x + 7$

Graph each function.

7. $y = x^2 - 2x + 4$
8. $y = -3x^2 + 18x$
9. $y = -2x^2 - 4x + 1$
10. $y = 2x^2 - 8x + 9$
11. $y = \frac{1}{3}x^2 + 2x + 7$
12. $y = x^2 + 6x + 9$
13. $y = x^2 + 3x + 6$
14. $y = -0.5x^2 + 4x - 3$
15. $y = -2x^2 - 8x - 1$

Extra Practice

Lesson 5-8

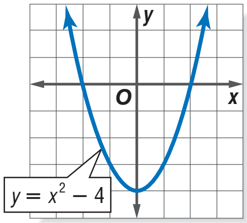
(pages 294–301)

Graph each inequality.

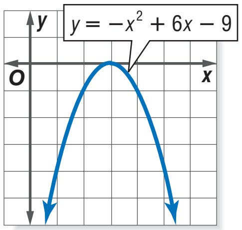
1. $y \leq 5x^2 + 3x - 2$
2. $y > -3x^2 + 2$
3. $y \geq x^2 - 8x$
4. $y \geq -x^2 - x + 3$
5. $y \leq 3x^2 + 4x - 8$
6. $y \leq -5x^2 + 2x - 3$
7. $y > 4x^2 + x$
8. $y \geq -x^2 - 3$

Use the graph of the related function of each inequality to write its solutions.

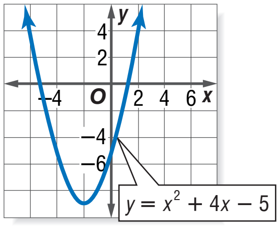
9. $x^2 - 4 \leq 0$



10. $-x^2 + 6x - 9 \geq 0$



11. $x^2 + 4x - 5 < 0$



Solve each inequality algebraically.

12. $x^2 - 1 < 0$
13. $10x^2 - x - 2 \geq 0$
14. $-x^2 - 5x - 6 > 0$
15. $-3x^2 \geq 5$
16. $x^2 - 2x - 8 \leq 0$
17. $2x^2 \geq 5x + 12$
18. $x^2 + 3x - 4 > 0$
19. $2x - x^2 \leq -15$

Lesson 6-1

(pages 312–318)

Simplify. Assume that no variable equals 0.

1. $x^7 \cdot x^3 \cdot x$
2. $m^8 \cdot m \cdot m^{10}$
3. $7^5 \cdot 7^2$
4. $(-3)^4(-3)$
5. $\dfrac{t^{12}}{t}$
6. $-\dfrac{16x^8}{8x^2}$
7. $\dfrac{6^5}{6^3}$
8. $\dfrac{p^5q^7}{p^2q^5}$
9. $-(m^3)^8$
10. $(3^5)^7$
11. -3^4
12. $(abc)^3$
13. $(x^2)^5$
14. $(b^4)^6$
15. $(-2y^5)^2$
16. $3x^0$
17. $(5x^4)^{-2}$
18. $(-3)^{-2}$
19. -3^{-2}
20. $\dfrac{x}{x^7}$
21. $-\left(\dfrac{x}{5}\right)^2$
22. $\left(\dfrac{5a^7}{2b^5c}\right)^3$
23. $\dfrac{1}{x^{-3}}$
24. $\dfrac{5^6a^{x+y}}{5^4a^{x-y}}$

Evaluate. Express the result in scientific notation.

25. $(8.95 \times 10^9)(1.82 \times 10^7)$
26. $(3.1 \times 10^5)(7.9 \times 10^{-8})$
27. $\dfrac{(2.38 \times 10^{13})(7.56 \times 10^{-5})}{(4.2 \times 10^{18})}$

Lesson 6-2

(pages 320–324)

Simplify.

1. $(4x^3 + 5x - 7x^2) + (-2x^3 + 5x^2 - 7y^2)$
2. $(2x^2 - 3x + 11) + (7x^2 + 2x - 8)$
3. $(-3x^2 + 7x + 23) + (-8x^2 - 5x + 13)$
4. $(-3x^2 + 7x + 23) - (-8x^2 - 5x + 13)$
5. $\dfrac{7}{uw}\left(4u^2w^3 - 5uw + \dfrac{w}{7u}\right)$
6. $-4x^5(-3x^4 - x^3 + x + 7)$
7. $(2x - 3)(4x + 7)$
8. $(3x - 5)(-2x - 1)$
9. $(3x - 5)(2x - 1)$
10. $(2x + 5)(2x - 5)$
11. $(3x - 7)(3x + 7)$
12. $(5 + 2w)(5 - 2w)$
13. $(2a^2 + 8)(2a^2 - 8)$
14. $(-5x + 10)(-5x - 10)$
15. $(4x - 3)^2$
16. $(5x + 6)^2$
17. $(-x + 1)^2$
18. $\dfrac{3}{4}x(x^2 + 4x + 14)$
19. $-\dfrac{1}{2}a^2(a^3 - 6a^2 + 5a)$

Lesson 6-3

(pages 325–330)

Find $p(5)$ and $p(-1)$ for each function.

1. $p(x) = 7x - 3$
2. $p(x) = -3x^2 + 5x - 4$
3. $p(x) = 5x^4 + 2x^2 - 2x$
4. $p(x) = -13x^3 + 5x^2$
5. $p(x) = x^6 - 2$
6. $p(x) = \frac{2}{3}x^2 + 5x$
7. $p(x) = x^3 + x^2 - x + 1$
8. $p(x) = x^4 - x^2 - 1$
9. $p(x) = 1 - x^3$

If $p(x) = -2x^2 + 5x + 1$ and $q(x) = x^3 - 1$, find each value.

10. $q(n)$
11. $p(2b)$
12. $q(z^3)$
13. $p(3m^2)$
14. $q(x + 1)$
15. $p(3 - x)$
16. $q(a^2 - 2)$
17. $3q(h - 3)$
18. $5[p(c - 4)]$
19. $q(n - 2) + q(n^2)$
20. $-3p(4a) - p(a)$
21. $2[q(d^2 + 1)] + 3q(d)$

Lesson 6-4

(pages 331–338)

For Exercises 1–16, complete each of the following.

a. **Graph each function by making a table of values.**

b. **Determine the values of x between which the real zeros are located.**

c. **Estimate the x-coordinates at which the relative maxima and relative minima occur.**

1. $f(x) = x^3 + x^2 - 3x$
2. $f(x) = -x^4 + x^3 + 5$
3. $f(x) = x^3 - 3x^2 + 8x - 7$
4. $f(x) = 2x^5 + 3x^4 - 8x^2 + x + 4$
5. $f(x) = x^4 - 5x^3 + 6x^2 - x - 2$
6. $f(x) = 2x^6 + 5x^4 - 3x^2 - 5$
7. $f(x) = -x^3 - 8x^2 + 3x - 7$
8. $f(x) = -x^4 - 3x^3 + 5x$
9. $f(x) = x^5 - 7x^4 - 3x^3 + 2x^2 - 4x + 9$
10. $f(x) = x^4 - 5x^3 + x^2 - x - 3$
11. $f(x) = x^4 - 128x^2 + 960$
12. $f(x) = -x^5 + x^4 - 208x^2 + 145x + 9$
13. $f(x) = x^5 - x^3 - x + 1$
14. $f(x) = x^3 - 2x^2 - x + 5$
15. $f(x) = 2x^4 - x^3 + x^2 - x + 1$
16. $f(x) = -x^3 - x^2 - x - 1$

Lesson 6-5

(pages 339–345)

Factor completely. If the polynomial is not factorable, write *prime*.

1. $14a^3b^3c - 21a^2b^4c + 7a^2b^3c$
2. $10ax - 2xy - 15ab + 3by$
3. $x^2 + x - 42$
4. $2x^2 + 5x + 3$
5. $6x^2 + 71x - 12$
6. $6x^4 - 12x^3 + 3x^2$
7. $x^2 - 6x + 2$
8. $x^2 - 2x - 15$
9. $6x^2 + 23x + 20$
10. $24x^2 - 76x + 40$
11. $6p^2 - 13pq - 28q^2$
12. $2x^2 - 6x + 3$
13. $x^2 + 49 - 14x$
14. $9x^2 - 64$
15. $36 - t^{10}$
16. $x^2 + 16$
17. $a^4 - 81b^4$
18. $3a^3 + 12a^2 - 63a$
19. $x^3 - 8x^2 + 15x$
20. $x^2 + 6x + 9$
21. $18x^3 - 8x$
22. $3x^2 - 42x + 40$
23. $2x^2 + 4x - 1$
24. $2x^3 + 6x^2 + x + 3$
25. $35ac - 3bd - 7ad + 15bc$
26. $5h^2 - 10hj + h - 2j$

Simplify. Assume that no denominator is equal to 0.

27. $\dfrac{x^2 + 8x + 15}{x^2 + 4x + 3}$
28. $\dfrac{x^2 + x - 2}{x^2 - 6x + 5}$
29. $\dfrac{x^2 - 15x + 56}{x^2 - 4x - 21}$
30. $\dfrac{x^2 + x - 6}{x^3 + 9x^2 + 27x + 27}$

Extra Practice

Lesson 6-6

(pages 349–355)

Simplify.

1. $\dfrac{18r^3s^2 + 36r^2s^3}{9r^2s^2}$
2. $\dfrac{15v^3w^2 - 5v^4w^3}{-5v^4w^3}$
3. $\dfrac{x^2 - x + 1}{x}$
4. $(5bh + 5ch) \div (b + c)$
5. $(25c^4d + 10c^3d^2 - cd) \div 5cd$
6. $(16f^{18} + 20f^9 - 8f^6) \div 4f^3$
7. $(33m^5 + 55mn^5 - 11m^3)(11m)^{-1}$
8. $(8g^3 + 19g^2 - 12g + 9) \div (g + 3)$
9. $(p^{21} + 3p^{14} + p^7 - 2)(p^7 + 2)^{-1}$
10. $(8k^2 - 56k + 98) \div (2k - 7)$
11. $(2r^2 + 5r - 3) \div (r + 3)$
12. $(n^3 + 125) \div (n + 5)$
13. $(10y^4 + 3y^2 - 7) \div (2y^2 - 1)$
14. $(q^4 + 8q^3 + 3q + 17) \div (q + 8)$
15. $(15v^3 + 8v^2 - 21v + 6) \div (5v - 4)$
16. $(-2x^3 + 15x^2 - 10x + 3) \div (x + 3)$
17. $(5s^3 + s^2 - 7) \div (s + 1)$
18. $(t^4 - 2t^3 + t^2 - 3t + 2) \div (t - 2)$
19. $(z^4 - 3z^3 - z^2 - 11z - 4) \div (z - 4)$
20. $(3r^4 - 6r^3 - 2r^2 + r - 6) \div (r + 1)$
21. $(2b^3 - 11b^2 + 12b + 9) \div (b - 3)$

Lesson 6-7

(pages 356–361)

Use synthetic substitution to find $f(3)$ and $f(-4)$ for each function.

1. $f(x) = x^2 - 6x + 2$
2. $f(x) = x^3 + 5x - 6$
3. $f(x) = x^3 - x^2 - 3x + 1$
4. $f(x) = -3x^3 + 5x^2 + 7x - 3$
5. $f(x) = 3x^5 - 5x^3 + 2x - 8$
6. $f(x) = 10x^3 + 2$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

7. $(x^3 - x^2 + x + 14); (x + 2)$
8. $(5x^3 - 17x^2 + 6x); (x - 3)$
9. $(2x^3 + x^2 - 41x + 20); (x - 4)$
10. $(x^3 - 8); (x - 2)$
11. $(x^2 + 6x + 5); (x + 1)$
12. $(x^4 + x^3 + x^2 + x); (x + 1)$
13. $(x^3 - 8x^2 + x + 42); (x - 7)$
14. $(x^4 + 5x^3 - 27x - 135); (x - 3)$
15. $(2x^3 - 15x^2 - 2x + 120); (2x + 5)$
16. $(6x^3 - 17x^2 + 6x + 8); (3x - 4)$
17. $(10x^3 + x^2 - 46x + 35); (5x - 7)$
18. $(x^3 + 9x^2 + 23x + 15); (x + 1)$

Lesson 6-8

(pages 362–368)

Solve each equation. State the number and type of roots.

1. $-5x - 7 = 0$
2. $3x^2 + 10 = 0$
3. $x^4 - 2x^3 = 23x^2 - 60x$

State the number of positive real zeros, negative real zeros, and imaginary zeros for each function.

4. $f(x) = 5x^8 - x^6 + 7x^4 - 8x^2 - 3$
5. $f(x) = 6x^5 - 7x^2 + 5$
6. $f(x) = -2x^6 - 5x^5 + 8x^2 - 3x + 1$
7. $f(x) = 4x^3 + x^2 - 38x + 56$
8. $f(x) = 3x^4 - 5x^3 + 2x^2 - 7x + 5$
9. $f(x) = x^5 - x^4 + 7x^3 - 25x^2 + 8x - 13$

Find all of the zeros of the function.

10. $f(x) = x^3 - 7x^2 + 16x - 10$
11. $f(x) = 10x^3 + 7x^2 - 82x + 56$
12. $f(x) = x^3 - 16x^2 + 79x - 114$
13. $f(x) = -3x^3 + 6x^2 + 5x - 8$
14. $f(x) = 24x^3 + 64x^2 + 6x - 10$
15. $f(x) = 2x^3 + 2x^2 - 34x + 30$

Lesson 6-9

(pages 369–373)

List all of the possible rational zeros for each function.

1. $f(x) = 3x^5 - 7x^3 - 8x + 6$
2. $f(x) = 4x^3 + 2x^2 - 5x + 8$
3. $f(x) = 6x^9 - 7$

Find all of the rational zeros for each function.

4. $f(x) = x^4 + 3x^3 - 7x^2 - 27x - 18$
5. $f(x) = 6x^4 - 31x^3 - 119x^2 + 214x + 560$
6. $f(x) = 20x^4 - 16x^3 + 11x^2 - 12x - 3$
7. $f(x) = 2x^4 - 30x^3 + 117x^2 - 75x + 280$
8. $f(x) = 3x^4 + 8x^3 + 9x^2 + 32x - 12$
9. $f(x) = x^5 - x^4 + x^3 + 3x^2 - x$

Find all of the zeros of each function.

10. $f(x) = x^4 + 8x^2 - 9$
11. $f(x) = 3x^4 - 9x^2 - 12$
12. $f(x) = 4x^4 + 19x^2 - 63$

Lesson 7-1

(pages 384–390)

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 3x + 5$, $g(x) = x - 3$
2. $f(x) = \sqrt{x}$, $g(x) = x^2$
3. $f(x) = x^2 - 5$, $g(x) = x^2 + 5$
4. $f(x) = x^2 + 1$, $g(x) = x + 1$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$, if they exist.

5. $f = \{(-1, 1), (2, -1), (-3, 5)\}$, $g = \{(1, -1), (-1, 2), (5, -3)\}$
6. $f = \{(0, 6), (5, -8), (-9, 2)\}$, $g = \{(-8, 3), (6, 4), (2, 1)\}$
7. $f = \{(8, 2), (6, 5), (-3, 4), (1, 0)\}$, $g = \{(2, 8), (5, 6), (4, -3), (0, 1)\}$
8. $f = \{(10, 4), (-1, 2), (5, 6), (-1, 0)\}$, $g = \{(-4, 10), (2, -9), (-7, 5), (-2, -1)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

9. $g(x) = 8 - 2x$, $h(x) = 3x$
10. $g(x) = x^2 - 7$, $h(x) = 3x + 2$
11. $g(x) = 2x + 7$, $h(x) = \frac{x - 7}{2}$
12. $g(x) = 3x + 2$, $h(x) = 5 - 3x$

If $f(x) = x^2 + 1$, $g(x) = 2x$, and $h(x) = x - 1$, find each value.

13. $g[f(1)]$
14. $[f \circ h](3)$
15. $[h \circ f](3)$
16. $[g \circ f](-2)$
17. $g[h(-20)]$
18. $f[h(-3)]$
19. $g[f(a)]$
20. $[f \circ (g \circ f)](c)$

Lesson 7-2

(pages 391–396)

Find the inverse of each relation.

1. $\{(-2, 7), (3, 0), (5, -8)\}$
2. $\{(-3, 9), (-2, 4), (3, 9), (-1, 1)\}$

Find the inverse of each function. Then graph the function and its inverse.

3. $f(x) = x - 7$
4. $y = 2x + 8$
5. $g(x) = 3x - 8$
6. $y = -5x - 6$
7. $y = -2$
8. $g(x) = 5 - 2x$
9. $h(x) = \frac{x}{5} + 1$
10. $h(x) = -\frac{2}{3}x$
11. $y = \frac{x - 5}{3}$
12. $y = \frac{1}{2}x - 1$
13. $f(x) = \frac{3x + 8}{4}$
14. $g(x) = \frac{2x - 1}{3}$

Determine whether each pair of functions are inverse functions.

15. $f(x) = \frac{2x - 3}{5}$, $g(x) = \frac{3x - 5}{3}$
16. $f(x) = 5x - 6$, $g(x) = \frac{x + 6}{5}$
17. $f(x) = 6 - 3x$, $g(x) = 2 - \frac{1}{3}x$
18. $f(x) = 3x - 7$, $g(x) = \frac{1}{3}x + 7$

Lesson 7-3

(pages 397–401)

Graph each function. State the domain and range of the function.

1. $y = \sqrt{x - 4}$
2. $y = \sqrt{x + 3} - 1$
3. $y = \frac{1}{3}\sqrt{x + 2}$
4. $y = \sqrt{2x + 5}$
5. $y = -\sqrt{4x}$
6. $y = 2\sqrt{x}$
7. $y = -3\sqrt{x}$
8. $y = \sqrt{x} + 5$
9. $y = \sqrt{2x} - 1$
10. $y = 5\sqrt{x} + 1$
11. $y = \sqrt{x + 1} - 2$
12. $y = 6 - \sqrt{x + 3}$

Graph each inequality.

13. $y > \sqrt{2x}$
14. $y \leq \sqrt{-5x}$
15. $y \geq \sqrt{x + 6} + 6$
16. $y < \sqrt{3x + 1} + 2$
17. $y \geq \sqrt{8x - 3} + 1$
18. $y < \sqrt{5x - 1} + 3$

Lesson 7-4

(pages 402–406)

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{289}$
2. $\sqrt{7832}$
3. $\sqrt[4]{0.0625}$
4. $\sqrt[3]{-343}$
5. $\sqrt[10]{32^4}$
6. $\sqrt[3]{49}$
7. $\sqrt[5]{5}$
8. $-\sqrt[4]{25}$

Simplify.

9. $\sqrt{9h^{22}}$
10. $\sqrt[5]{0}$
11. $\sqrt{\frac{16}{9}}$
12. $\sqrt{\left(-\frac{2}{3}\right)^4}$
13. $\sqrt[5]{-32}$
14. $-\sqrt{-144}$
15. $\sqrt[4]{a^{16}b^8}$
16. $\pm\sqrt[4]{81x^4}$
17. $\sqrt[5]{\frac{1}{100{,}000}}$
18. $\sqrt[3]{-d^6}$
19. $\sqrt[5]{p^{25}q^{15}r^5s^{20}}$
20. $\sqrt[4]{(2x^2 - y^8)^8}$
21. $\pm\sqrt{16m^6n^2}$
22. $-\sqrt[3]{(2x - y)^3}$
23. $\sqrt[4]{(r + s)^4}$
24. $\sqrt{9a^2 + 6a + 1}$
25. $\sqrt{4y^2 + 12y + 9}$
26. $-\sqrt{x^2 - 2x + 1}$
27. $\pm\sqrt{x^2 + 2x + 1}$
28. $\sqrt[3]{a^3 + 6a^2 + 12a + 8}$

Lesson 7-5

(pages 408–414)

Simplify.

1. $\sqrt{75}$
2. $7\sqrt{12}$
3. $\sqrt[3]{81}$
4. $\sqrt{5r^5}$
5. $\sqrt[4]{7^8x^5y^6}$
6. $3\sqrt{5} + 6\sqrt{5}$
7. $\sqrt{18} - \sqrt{50}$
8. $4\sqrt[3]{32} + \sqrt[3]{500}$
9. $\sqrt{12}\sqrt{27}$
10. $3\sqrt{12} + 2\sqrt{300}$
11. $\sqrt[3]{54} - \sqrt[3]{24}$
12. $\sqrt{10}(2 - \sqrt{5})$
13. $-\sqrt{3}(2\sqrt{6} - \sqrt{63})$
14. $(5 + \sqrt{2})(3 + \sqrt{3})$
15. $(2 + \sqrt{5})(2 - \sqrt{5})$
16. $\left(8 + \sqrt{11}\right)^2$
17. $\left(\sqrt{3} + \sqrt{6}\right)\left(\sqrt{3} - \sqrt{6}\right)$
18. $\left(\sqrt{8} + \sqrt{13}\right)^2$
19. $\left(1 - \sqrt{7}\right)\left(4 + \sqrt{7}\right)$
20. $\left(5 - 2\sqrt{7}\right)^2$
21. $\sqrt{\frac{3m^3}{24n^5}}$
22. $\frac{\sqrt{18}}{\sqrt{32}}$
23. $2\sqrt[3]{\frac{r^5}{2s^2t}}$
24. $\sqrt[3]{\frac{4}{7}}$
25. $\sqrt[5]{\frac{32}{a^4}}$
26. $\sqrt{\frac{2}{3}} - \sqrt{\frac{3}{8}}$
27. $\frac{5}{3 - \sqrt{10}}$
28. $\frac{\sqrt{5}}{1 + \sqrt{3}}$
29. $\frac{-2 + \sqrt{7}}{2 + \sqrt{7}}$
30. $\frac{1 - \sqrt{3}}{1 + \sqrt{8}}$
31. $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$
32. $\frac{x + \sqrt{5}}{x - \sqrt{5}}$

Lesson 7-6

(pages 415–421)

Write each expression in radical form.

1. $10^{\frac{1}{3}}$
2. $8^{\frac{1}{4}}$
3. $a^{\frac{2}{3}}$
4. $(b^2)^{\frac{3}{4}}$

Write each radical using rational exponents.

5. $\sqrt{35}$
6. $\sqrt[4]{32}$
7. $3\sqrt{27a^2x}$
8. $\sqrt[5]{25ab^3c^4}$

Evaluate each expression.

9. $2401^{\frac{1}{4}}$
10. $27^{\frac{4}{3}}$
11. $(-32)^{\frac{2}{5}}$
12. $-81^{\frac{3}{4}}$
13. $(-125)^{-\frac{2}{3}}$
14. $16^{\frac{5}{2}} \cdot 16^{\frac{1}{2}}$
15. $8^{-\frac{2}{3}} \cdot 64^{\frac{1}{6}}$
16. $\left(\frac{48}{1875}\right)^{-\frac{5}{4}}$

Simplify each expression.

17. $7^{\frac{5}{9}} \cdot 7^{\frac{4}{9}}$
18. $32^{\frac{2}{3}} \cdot 32^{\frac{3}{5}}$
19. $\left(k^{\frac{8}{5}}\right)^5$
20. $x^{\frac{2}{5}} \cdot x^{\frac{8}{5}}$
21. $m^{\frac{2}{5}} \cdot m^{\frac{4}{5}}$
22. $\left(p^{\frac{5}{4}} \cdot q^{\frac{7}{2}}\right)^{\frac{8}{3}}$
23. $\left(4^{\frac{9}{2}}c^{\frac{3}{2}}\right)^2$
24. $\dfrac{7^{\frac{3}{4}}}{7^{\frac{5}{3}}}$
25. $\dfrac{1}{t^{\frac{9}{5}}}$
26. $a^{-\frac{8}{7}}$
27. $\dfrac{r}{r^{\frac{7}{5}}}$
28. $\sqrt[4]{36}$
29. $\sqrt[4]{9a^2}$
30. $\sqrt[3]{\sqrt{81}}$
31. $\dfrac{v^{\frac{11}{7}} - v^{\frac{4}{7}}}{v^{\frac{4}{7}}}$
32. $\dfrac{1}{5^{\frac{1}{2}} + 3^{\frac{1}{2}}}$

Lesson 7-7

(pages 422–427)

Solve each equation or inequality.

1. $\sqrt{x} = 16$
2. $\sqrt{z+3} = 7$
3. $\sqrt[3]{a+5} = 1$
4. $5\sqrt{s} - 8 = 3$
5. $\sqrt[4]{m+7} + 11 = 9$
6. $d + \sqrt{d^2 - 8} = 4$
7. $g\sqrt{5} + 4 = g + 4$
8. $\sqrt{x-8} = \sqrt{13+x}$
9. $\sqrt{3x+9} > 2$
10. $\sqrt{3n-1} \leq 5$
11. $2 - 4\sqrt{21-6c} < -6$
12. $\sqrt{5y+4} > 8$
13. $\sqrt{2w+3} + 5 \geq 7$
14. $\sqrt{2c+3} - 7 > 0$
15. $\sqrt{3z-5} - 3 = 1$
16. $\sqrt{5y+1} + 6 < 10$
17. $\sqrt{3n+1} - 2 \leq 6$
18. $\sqrt{y-5} - \sqrt{y} \geq 1$
19. $(5n-1)^{\frac{1}{2}} = 0$
20. $(7x-6)^{\frac{1}{3}} + 1 = 3$
21. $(6a-8)^{\frac{1}{4}} + 9 \geq 10$

Lesson 8-1

(pages 442–449)

Simplify each expression.

1. $\dfrac{25xy^2}{15y}$
2. $\dfrac{-4a^2b^3}{28ab^4}$
3. $\dfrac{(-2cd^3)^2}{8c^2d^5}$
4. $\dfrac{3x^3}{-2} \cdot \dfrac{-4}{9x}$
5. $\dfrac{21x^2}{-5} \cdot \dfrac{10}{7x^3}$
6. $\dfrac{2u^2}{3} \div \dfrac{6u^3}{5}$
7. $\dfrac{15x^3}{14} \div \dfrac{18x}{7}$
8. $\dfrac{xy^2}{2} \cdot \dfrac{x^2}{2y} \cdot \dfrac{2}{x^2y}$
9. $axy \div \dfrac{ax}{y}$
10. $\dfrac{9u^2}{28v} \div \dfrac{27u^2}{8v^2}$
11. $\dfrac{x^2-4}{4x^2-1} \cdot \dfrac{2x-1}{x+2}$
12. $\dfrac{x^2-1}{2x^2-x-1} \div \dfrac{x^2-4}{2x^2-3x-2}$
13. $\dfrac{2x^2+x-1}{2x^2+3x-2} \div \dfrac{x^2-2x+1}{x^2+x-2}$
14. $\dfrac{\frac{(ab)^2}{c}}{\frac{xa^3b}{cx^2}}$
15. $\dfrac{x^4-y^4}{x^3+y^3} \div \dfrac{x^3-y^3}{x+y}$

Lesson 8-2

(pages 450–456)

Find the LCM of each set of polynomials.

1. $2a^2b, 4ab^2, 20a$
2. $x^2 - 4x - 12, x^2 + 7x + 10$

Simplify each expression.

3. $\frac{12}{7d} - \frac{3}{14d}$
4. $\frac{x+1}{x} - \frac{x-1}{x^2}$
5. $\frac{2x+1}{4x^2} - \frac{x+3}{6x}$
6. $\frac{7x}{13y^2} + \frac{4y}{6x^2}$
7. $\frac{x}{x-1} + \frac{1}{1-x}$
8. $\frac{1}{3v^2} + \frac{1}{uv} + \frac{3}{4u^2}$
9. $\frac{1}{x^2-x} + \frac{1}{x^2+x}$
10. $\frac{1}{x^2-1} - \frac{1}{(x-1)^2}$
11. $\frac{5}{x} - \frac{3}{x+5}$
12. $y - 1 + \frac{1}{y-1}$
13. $3m + 1 - \frac{2m}{3m+1}$
14. $\frac{3x}{x-y} + \frac{4x}{y-x}$
15. $\frac{4}{a^2-4} - \frac{3}{a^2+4a+4}$
16. $\frac{4}{3-3z^2} - \frac{2}{z^2+5z+4}$
17. $\frac{2c}{c^2-9} - \frac{1}{c^2+6c+9}$
18. $\dfrac{\frac{1}{x+y}}{\frac{1}{x}+\frac{1}{y}}$
19. $\dfrac{1-\frac{1}{x+1}}{1+\frac{1}{x-1}}$
20. $\dfrac{4+\frac{1}{x-2}}{3-\frac{1}{x-2}}$

Lesson 8-3

(pages 457–463)

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{1}{x+4}$
2. $f(x) = \frac{x-2}{x+3}$
3. $f(x) = \frac{5}{(x+1)(x-8)}$
4. $f(x) = \frac{x}{x+2}$
5. $f(x) = \frac{x^2-4}{x+2}$
6. $f(x) = \frac{x^2+x-6}{x^2+8x+15}$

Graph each rational function.

7. $f(x) = \frac{1}{x-5}$
8. $f(x) = \frac{3x}{x+1}$
9. $f(x) = \frac{x^2-16}{x-4}$
10. $f(x) = \frac{x}{x-6}$
11. $f(x) = \frac{1}{(x-3)^2}$
12. $f(x) = \frac{2}{(x+3)(x-4)}$
13. $f(x) = \frac{x+4}{x^2-1}$
14. $f(x) = \frac{x+2}{x+3}$
15. $f(x) = \frac{x^2+5x-14}{x^2+9x+14}$

Lesson 8-4

(pages 465–471)

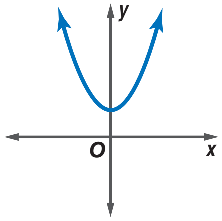
State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

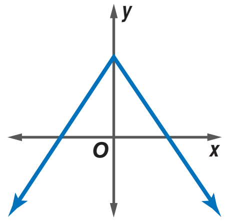
1. $xy = 10$
2. $\frac{x}{7} = y$
3. $\frac{x}{y} = -6$
4. $10x = y$
5. $x = \frac{2}{y}$
6. $A = \ell w$
7. $\frac{1}{4}b = -\frac{3}{5}c$
8. $D = rt$
9. If y varies directly as x and $y = 16$ when $x = 4$, find y when $x = 12$.
10. If x varies inversely as y and $x = 12$ when $y = -3$, find x when $y = -18$.
11. If m varies directly as w and $m = -15$ when $w = 2.5$, find m when $w = 12.5$.
12. If y varies jointly as x and z and $y = 10$ when $z = 4$ and $x = 5$, find y when $x = 4$ and $z = 2$.
13. If y varies inversely as x and $y = \frac{1}{4}$ when $x = 24$, find y when $x = \frac{3}{4}$.

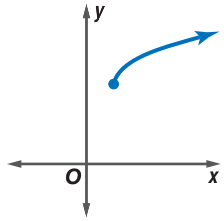
Lesson 8-5

(pages 473–478)

Identify the type of function represented by each graph.

1. 

2. 

3. 

Identify the function represented by each equation. Then graph the equation.

4. $y = \sqrt{5x}$ **5.** $y = \frac{3}{4}x$ **6.** $y = |x| + 3$

7. $y = x^2 - 2$ **8.** $y = \frac{2}{x}$ **9.** $y = 2[\![x]\!]$

10. $y = -2x^2 + 1$ **11.** $y = \frac{x^2 + 2x - 3}{x^2 + 7x + 12}$ **12.** $y = -3$

Lesson 8-6

(pages 479–486)

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-3} = \frac{1}{4}$ **2.** $\frac{5}{x} + \frac{3}{5} = \frac{2}{x}$ **3.** $\frac{5}{b-2} < 5$

4. $\frac{4}{a+3} > 2$ **5.** $\frac{x-2}{x} = \frac{x-4}{x-6}$ **6.** $-6 - \frac{8}{n} < n$

7. $\frac{2}{d} + \frac{1}{d-2} = 1$ **8.** $\frac{1}{2+3x} + \frac{2}{2-3x} = 0$ **9.** $\frac{1}{n+1} + \frac{1}{n-1} = \frac{2}{n^2-1}$

10. $\frac{p}{p+1} + \frac{3}{p-3} + 1 = 0$ **11.** $\frac{5z+2}{z^2-4} = \frac{-5z}{2-z} + \frac{2}{z+2}$ **12.** $\frac{1}{x-3} + \frac{2}{x^2-9} = \frac{5}{x+3}$

13. $\frac{1}{m^2-1} = \frac{2}{m^2+m-2}$ **14.** $\frac{12}{x^2-16} - \frac{24}{x-4} = 3$ **15.** $n + \frac{1}{n+3} = \frac{n^2}{n-1}$

Lesson 9-1

(pages 498–506)

Sketch the graph of each function. Then state the function's domain and range.

1. $y = 3(5)^x$ **2.** $y = 0.5(2)^x$ **3.** $y = 3\left(\frac{1}{4}\right)^x$ **4.** $y = 2(1.5)^x$

Determine whether each function represents exponential *growth* or *decay*.

5. $y = 4(3)^x$ **6.** $y = 10^{-x}$ **7.** $y = 5\left(\frac{1}{2}\right)^x$ **8.** $y = 2\left(\frac{5}{4}\right)^x$

Write an exponential function for the graph that passes through the given points.

9. (0, 6) and (2, 54) **10.** (0, –4) and (–4, –64) **11.** (0, 1.5) and (3, 40.5)

Solve each equation or inequality. Check your solution.

12. $27^{2x-1} = 3$ **13.** $8^{2+x} \geq 2$ **14.** $4^{2x+5} < 8^{x+1}$ **15.** $6^{x+1} = 36^{x-1}$

16. $10^{x-1} > 100^{4-x}$ **17.** $\left(\frac{1}{5}\right)^{x-3} = 125$ **18.** $2^{x^2+1} = 32$ **19.** $36^x = 6^{x^2-3}$

Lesson 9-2
(pages 509–517)

Write each equation in logarithmic form.

1. $3^5 = 243$
2. $10^3 = 1000$
3. $4^{-3} = \frac{1}{64}$

Write each equation in exponential form.

4. $\log_2 \frac{1}{8} = -3$
5. $\log_{25} 5 = \frac{1}{2}$
6. $\log_7 \frac{1}{7} = -1$

Evaluate each expression.

7. $\log_4 16$
8. $\log_{10} 10{,}000$
9. $\log_3 \frac{1}{9}$
10. $\log_2 1024$
11. $\log_6 6^5$
12. $\log_{\frac{1}{2}} 8$
13. $\log_{11} 121$
14. $5^{\log_5 10}$

Solve each equation or inequality. Check your solutions.

15. $\log_8 b = 2$
16. $\log_4 x < 3$
17. $\log_{\frac{1}{9}} n = -\frac{1}{2}$
18. $\log_x 7 = 1$
19. $\log_{\frac{2}{3}} a < 3$
20. $\log_2 (x^2 - 9) = 4$

Lesson 9-3
(pages 520–526)

Use $\log_3 5 \approx 1.4651$ and $\log_3 7 \approx 1.7712$ to approximate the value of each expression.

1. $\log_3 \frac{7}{5}$
2. $\log_3 245$
3. $\log_3 35$

Solve each equation. Check your solutions.

4. $\log_2 x + \log_2 (x - 2) = \log_2 3$
5. $\log_3 x = 2 \log_3 3 + \log_3 5$
6. $\log_5 (x^2 + 7) = \frac{2}{3} \log_5 64$
7. $\log_2 (x^2 - 9) = 4$
8. $\log_3 (x + 2) + \log_3 6 = 3$
9. $\log_6 x + \log_6 (x - 5) = 2$
10. $\log_5 (x + 3) = \log_5 8 - \log_5 2$
11. $2 \log_3 x - \log_3 (x - 2) = 2$
12. $\log_6 x = \frac{3}{2} \log_6 9 + \log_6 2$
13. $\log_8 (x + 6) + \log_8 (x - 6) = 2$
14. $\log_3 14 + \log_3 x = \log_3 42$
15. $\log_{10} x = \frac{1}{2} \log_{10} 81$

Lesson 9-4
(pages 528–533)

Use a calculator to evaluate each expression to four decimal places.

1. $\log 55$
2. $\log 6.7$
3. $\log 3.3$
4. $\log 0.08$
5. $\log 9.9$
6. $\log 0.6$

Solve each equation or inequality. Round to four decimal places.

7. $2^x = 15$
8. $4^{2a} > 45$
9. $7^{2x} = 35$
10. $11^{x + 4} > 57$
11. $1.5^{a - 7} = 9.6$
12. $3^{b^2} = 64$
13. $7^{3c} < 35^{2c - 1}$
14. $5^{m^2 + 1} = 30$
15. $7^{3y - 1} < 2^{2y + 4}$
16. $9^{n - 3} = 2^{n + 3}$
17. $11^{t + 1} \leq 22^{t + 3}$
18. $2^{3a - 1} = 3^{a + 2}$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

19. $\log_3 21$
20. $\log_4 62$
21. $\log_5 28$
22. $\log_2 25$

Lesson 9-5

(pages 536–542)

Use a calculator to evaluate each expression to four decimal places.

1. e^3
2. $e^{0.75}$
3. e^{-4}
4. $e^{-2.5}$
5. $\ln 5$
6. $\ln 8$
7. $\ln 8.4$
8. $\ln 0.6$

Write an equivalent exponential or logarithmic equation.

9. $e^x = 10$
10. $\ln x \approx 2.3026$
11. $e^3 = 9x$
12. $\ln 0.2 = x$

Solve each equation or inequality.

13. $25e^x = 1000$
14. $e^{0.075x} > 25$
15. $e^x < 3.8$
16. $-2e^x + 5 = 1$
17. $5 + 4e^{2x} = 17$
18. $e^{-3x} \leq 15$
19. $\ln 7x = 10$
20. $\ln 4x = 8$
21. $3 \ln 2x \geq 9$
22. $\ln (x + 2) = 4$
23. $\ln (2x + 3) > 0$
24. $\ln (3x - 1) = 5$

Lesson 9-6

(pages 544–550)

1. **FARMING** Mr. Rogers purchased a combine for $175,000 for his farming operation. It is expected to depreciate at a rate of 18% per year. What will be the value of the combine in 3 years?
2. **REAL ESTATE** The Jacksons bought a house for $65,000 in 1992. Houses in the neighborhood have appreciated at the rate of 4.5% a year. How much is the house worth in 2003?
3. **POPULATION** In 1950, the population of a city was 50,000. Since then, the population has increased by 2.25% per year. If it continues to grow at this rate, what will the population be in 2005?
4. **BEARS** In a particular state, the population of black bears has been decreasing at the rate of 0.75% per year. In 1990, it was estimated that there were 400 black bears in the state. If the population continues to decline at the same rate, what will the population be in 2010?

Lesson 10-1

(pages 562–566)

Find the midpoint of the line segment with endpoints at the given coordinates.

1. $(7, -3), (-11, 13)$
2. $(16, 29), (-7, 2)$
3. $(43, -18), (-78, -32)$
4. $(-7.54, 3.42), (4.89, -9.28)$
5. $\left(\frac{1}{2}, \frac{1}{4}\right), \left(\frac{2}{3}, \frac{3}{5}\right)$
6. $\left(-\frac{1}{4}, \frac{2}{3}\right), \left(-\frac{1}{2}, -\frac{1}{2}\right)$

Find the distance between each pair of points with the given coordinates.

7. $(5, 7), (3, 19)$
8. $(-2, -1), (5, 3)$
9. $(-3, 15), (7, -8)$
10. $(6, -3), (-4, -9)$
11. $(3.89, -0.38), (4.04, -0.18)$
12. $(5\sqrt{3}, 2\sqrt{2}), (-11\sqrt{3}, -4\sqrt{2})$
13. $\left(\frac{1}{4}, 0\right), \left(-\frac{2}{3}, \frac{1}{2}\right)$
14. $\left(4, -\frac{5}{6}\right), \left(-2, \frac{1}{6}\right)$
15. A circle has a radius with endpoints at $(-3, 1)$ and $(2, -5)$. Find the circumference and area of the circle. Write the answer in terms of π.
16. Triangle ABC has vertices $A(0, 0)$, $B(-3, 4)$, and $C(2, 6)$. Find the perimeter of the triangle.

Lesson 10-2

(pages 567–573)

Write each equation in standard form.

1. $y = x^2 - 4x + 7$
2. $y = 2x^2 + 12x + 17$
3. $x = 3y^2 - 6y + 5$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. $y + 4 = x^2$
5. $y = 5(x + 2)^2$
6. $4(y + 2) = 3(x - 1)^2$
7. $5x + 3y^2 = 15$
8. $y = 2x^2 - 8x + 7$
9. $x = 2y^2 - 8y + 7$
10. $3(x - 8)^2 = 5(y + 3)$
11. $x = 3(y + 4)^2 + 1$
12. $8y + 5x^2 + 30x + 101 = 0$
13. $x = -\frac{1}{5}y^2 + \frac{8}{5}y - 7$
14. $6x = y^2 - 6y + 39$
15. $-8y = x^2$
16. $y = 4x^2 + 24x + 38$
17. $y = x^2 - 6x + 3$
18. $y = x^2 + 4x + 1$

Write an equation for each parabola described below. Then graph.

19. focus (1, 1), directrix $y = -1$
20. vertex (−1, 2), directrix $y = -4$

Lesson 10-3

(pages 574–579)

Write an equation for the circle that satisfies each set of conditions.

1. center (3, 2), $r = 5$ units
2. center (−5, 8), $r = 3$ units
3. center (1, −6), $r = \frac{2}{3}$ units
4. center (0, 7), tangent to x-axis
5. center (−2, −4), tangent to y-axis
6. endpoints of a diameter at (−9, 0) and (2, −5)
7. endpoints of a diameter at (4, 1) and (−3, 2)
8. center (6, −10), passes through origin
9. center (0.8, 0.5), passes through (2, 2)

Find the center and radius of the circle with the given equation. Then graph.

10. $x^2 + y^2 = 36$
11. $(x - 5)^2 + (y + 4)^2 = 1$
12. $x^2 + 3x + y^2 - 5y = 0.5$
13. $x^2 + y^2 = 14x - 24$
14. $x^2 + y^2 = 2(y - x)$
15. $x^2 + 10x + \left(y - \sqrt{3}\right)^2 = 11$
16. $x^2 + y^2 = 4x + 9$
17. $x^2 + y^2 - 6x + 4y = 156$
18. $x^2 + y^2 - 2x + 7y = 1$

Lesson 10-4

(pages 581–588)

Write an equation for the ellipse that satisfies each set of conditions.

1. endpoints of major axis at (−2, 7) and (4, 7), endpoints of minor axis at (1, 5) and (1, 9)
2. endpoints of minor axis at (1, −4) and (1, 5), endpoints of major axis at (−4, 0.5) and (6, 0.5)
3. major axis 24 units long and parallel to the y-axis, minor axis 4 units long, center at (0, 3)

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

4. $\frac{x^2}{36} + \frac{y^2}{81} = 1$
5. $\frac{x^2}{121} + \frac{(y-5)^2}{16} = 1$
6. $\frac{(x+2)^2}{12} + \frac{(y+1)^2}{16} = 1$
7. $8x^2 + 2y^2 = 32$
8. $7x^2 + 3y^2 = 84$
9. $9x^2 + 16y^2 = 144$
10. $169x^2 - 338x + 169 + 25y^2 = 4225$
11. $x^2 + 4y^2 + 8x - 64y = -128$
12. $4x^2 + 5y^2 = 6(6x + 5y) + 658$
13. $9x^2 + 16y^2 - 54x + 64y + 1 = 0$

Lesson 10-5

(pages 590–597)

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

1. $\frac{y^2}{25} - \frac{x^2}{9} = 1$
2. $\frac{x^2}{4} - \frac{y^2}{9} = 1$
3. $\frac{x^2}{81} - \frac{y^2}{36} = 1$
4. $\frac{(x-4)^2}{64} - \frac{(y+1)^2}{16} = 1$
5. $\frac{(y-7)^2}{2.25} - \frac{(x-3)^2}{4} = 1$
6. $(x+5)^2 - \frac{(y+3)^2}{48} = 1$
7. $x^2 - 9y^2 = 36$
8. $4x^2 - 9y^2 = 72$
9. $49x^2 - 16y^2 = 784$
10. $576y^2 = 49x^2 + 490x + 29{,}449$
11. $25(y+5)^2 - 20(x-1)^2 = 500$

Write an equation for the hyperbola that satisfies each set of conditions.

12. vertices $(-3, 0)$ and $(3, 0)$; conjugate axis of length 8 units
13. vertices $(0, -7)$ and $(0, 7)$; conjugate axis of length 25 units
14. center $(0, 0)$; horizontal transverse axis of length 12 units and a conjugate axis of length 10 units

Lesson 10-6

(pages 598–602)

Write each equation in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equation.

1. $9x^2 - 36x + 36 = 4y^2 + 24y + 72$
2. $x^2 + 4x + 2y^2 + 16y + 32 = 0$
3. $x^2 + 6x + y^2 - 6y + 9 = 0$
4. $9y^2 = 25x^2 + 400x + 1825$
5. $2y^2 + 12y - x + 6 = 0$
6. $x^2 + y^2 = 10x + 2y + 23$
7. $3x^2 + y = 12x - 17$
8. $9x^2 - 18x + 16y^2 + 160y = -265$
9. $x^2 + 10x + 5 = 4y^2 + 16$
10. $\frac{(y-5)^2}{4} - (x+1)^2 = 4$
11. $9x^2 + 49y^2 = 441$
12. $4x^2 - y^2 = 4$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola, circle, ellipse,* or *hyperbola*.

13. $(x+3)^2 = 8(y+2)$
14. $x^2 + 4x + y^2 - 8y = 2$
15. $2x^2 - 13y^2 + 5 = 0$
16. $16(x-3)^2 + 81(y+4)^2 = 1296$

Lesson 10-7

(pages 603–608)

Solve each system of inequalities by graphing.

1. $\frac{x^2}{16} - \frac{y^2}{1} \geq 1$
$x^2 + y^2 \leq 49$
2. $\frac{x^2}{25} + \frac{y^2}{16} \leq 1$
$y \leq x - 2$
3. $y \geq x + 3$
$x^2 + y^2 < 25$
4. $4x^2 + (y-3)^2 \leq 16$
$x + 2y \geq 4$

Find the exact solution(s) of each system of equations.

5. $\frac{x^2}{16} + \frac{y^2}{16} = 1$
$y = x + 3$
6. $x = y^2$
$(x+3)^2 + y^2 = 53$
7. $\frac{x^2}{3} - \frac{(y+2)^2}{4} = 1$
$x^2 = y^2 + 11$
8. $\frac{(x-1)^2}{5} + \frac{y^2}{2} = 1$
$y = x + 1$
9. $x^2 + y^2 = 13$
$x^2 - y^2 = -5$
10. $\frac{x^2}{25} - \frac{y^2}{5} = 1$
$y = x - 4$
11. $x^2 + y = 0$
$x + y = -2$
12. $x^2 - 9y^2 = 36$
$x = y$
13. $4x^2 + 6y^2 = 360$
$y = x$

Extra Practice

Lesson 11-1
(pages 622–628)

Find the next four terms of each arithmetic sequence.

1. 9, 7, 5, . . .
2. 3, 4.5, 6, . . .
3. 40, 35, 30, . . .
4. 2, 5, 8, . . .

Find the first five terms of each arithmetic sequence described.

5. $a_1 = 1, d = 7$
6. $a_1 = -5, d = 2$
7. $a_1 = 1.2, d = 3.7$
8. $a_1 = -\frac{5}{4}, d = -\frac{1}{2}$

Find the indicated term of each arithmetic sequence.

9. $a_1 = 4, d = 5, n = 10$
10. $a_1 = -30, d = -6, n = 5$
11. $a_1 = -3, d = 32, n = 8$

Write an equation for the *n*th term of each arithmetic sequence.

12. 3, 5, 7, 9, . . .
13. 2, −1, −4, −7, . . .
14. 20, 28, 36, 44, . . .

Find the arithmetic means in each sequence.

15. 2, __?__, __?__, __?__, 34
16. 0, __?__, __?__, __?__, −28
17. −10, __?__, __?__, __?__, 14

Lesson 11-2
(pages 629–635)

Find S_n for each arithmetic series described.

1. $a_1 = 3, a_n = 20, n = 6$
2. $a_1 = 90, a_n = -4, n = 10$
3. $a_1 = 16, a_n = 14, n = 12$
4. $a_1 = -1, d = 10, n = 30$
5. $a_1 = 4, d = -5, n = 11$
6. $a_1 = 5, d = -\frac{1}{2}, n = 17$

Find the sum of each arithmetic series.

7. $\sum_{n=1}^{6}(n + 2)$
8. $\sum_{n=5}^{10}(2n - 5)$
9. $\sum_{k=1}^{5}(40 - 2k)$
10. $\sum_{k=8}^{12}(6 - 3k)$
11. $\sum_{n=1}^{4}(10n + 2)$
12. $\sum_{n=6}^{10}(2 + 3n)$

Find the first three terms of each arithmetic series described.

13. $a_1 = 11, a_n = 38, S_n = 245$
14. $n = 12, a_n = 13, S_n = -42$
15. $n = 11, a_n = 5, S_n = 0$

Lesson 11-3
(pages 636–641)

Find the next two terms of each geometric sequence.

1. 5, 15, 45, . . .
2. 2, 10, 50, . . .
3. 64, 16, 4, . . .
4. −9, 27, −81, . . .
5. 0.5, 0.75, 1.125, . . .
6. $\frac{1}{2}, -\frac{3}{8}, \frac{9}{32}, \ldots$

Find the first five terms of each geometric sequence described.

7. $a_1 = -2, r = 6$
8. $a_1 = 4, r = -5$
9. $a_1 = 0.8, r = 2.5$
10. $a_1 = -\frac{1}{3}, r = -\frac{3}{5}$

Find the indicated term of each geometric sequence.

11. $a_1 = 5, r = 7, n = 6$
12. $a_1 = 200, r = -\frac{1}{2}, n = 10$
13. $a_1 = 60, r = -2, n = 4$

Write an equation for the *n*th term of each geometric sequence.

14. 20, 40, 80, . . .
15. $-\frac{1}{2}, -\frac{1}{8}, -\frac{1}{32}, \ldots$

Find the geometric means in each sequence.

16. 1, __?__, __?__, __?__, 81
17. 5, __?__, __?__, __?__, 6480

Extra Practice

Lesson 11-4

(pages 643–649)

Find S_n for each geometric series described.

1. $a_1 = \frac{1}{81}, r = 3, n = 6$
2. $a_1 = 1, r = -2, n = 7$
3. $a_1 = 5, r = 4, n = 5$
4. $a_1 = -27, r = -\frac{1}{3}, n = 6$
5. $a_1 = 1000, r = \frac{1}{2}, n = 7$
6. $a_1 = 125, r = -\frac{2}{5}, n = 5$
7. $a_1 = 10, r = 3, n = 6$
8. $a_1 = 1250, r = -\frac{1}{5}, n = 5$
9. $a_1 = 1215, r = \frac{1}{3}, n = 5$
10. $a_1 = 16, r = \frac{3}{2}, n = 5$
11. $a_1 = 7, r = 2, n = 7$
12. $a_1 = -\frac{3}{2}, r = -\frac{1}{2}, n = 6$

Find the sum of each geometric series.

13. $\sum_{k=1}^{5} 2^k$
14. $\sum_{n=0}^{3} 3^{-n}$
15. $\sum_{n=0}^{3} 2(5^n)$
16. $\sum_{k=2}^{5} -(-3)^{k-1}$

Find the indicated term for each geometric series described.

17. $S_n = 300, a_n = 160, r = 2; a_1$
18. $S_n = -171, n = 9, r = -2; a_5$
19. $S_n = -4372, a_n = -2916, r = 3; a_4$

Lesson 11-5

(pages 650–655)

Find the sum of each infinite geometric series, if it exists.

1. $a_1 = 54, r = \frac{1}{3}$
2. $a_1 = 2, r = -1$
3. $a_1 = 1000, r = -0.2$
4. $a_1 = 7, r = \frac{3}{7}$
5. $49 + 14 + 4 + \ldots$
6. $\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \ldots$
7. $12 - 4 + \frac{4}{3} - \ldots$
8. $3 - 9 + 27 - \ldots$
9. $3 - 2 + \frac{4}{3} - \ldots$
10. $\sum_{n=1}^{\infty} 3\left(\frac{1}{4}\right)^{n-1}$
11. $\sum_{n=1}^{\infty} 5\left(-\frac{1}{10}\right)^{n-1}$
12. $\sum_{n=1}^{\infty} -\frac{2}{3}\left(-\frac{3}{4}\right)^{n-1}$

Write each repeating decimal as a fraction.

13. $0.\overline{4}$
14. $0.\overline{27}$
15. $0.\overline{123}$
16. $0.\overline{645}$
17. $0.\overline{67}$
18. $0.8\overline{53}$

Lesson 11-6

(pages 658–662)

Find the first five terms of each sequence.

1. $a_1 = 4, a_{n+1} = 2a_{n+1}$
2. $a_1 = 6, a_{n+1} = a_n + 7$
3. $a_1 = 16, a_{n+1} = a_n + (n + 4)$
4. $a_1 = 1, a_{n+1} = \frac{n}{n+2} \cdot a_n$
5. $a_1 = -\frac{1}{2}, a_{n+1} = 2a_n + \frac{1}{4}$
6. $a_1 = \frac{1}{3}, a_2 = \frac{1}{4}, a_{n+1} = a_n + a_{n-1}$

Find the first three iterates of each function for the given initial value.

7. $f(x) = 3x - 1, x_0 = 3$
8. $f(x) = 2x^2 - 8, x_0 = -1$
9. $f(x) = 4x + 5, x_0 = 0$
10. $f(x) = 3x^2 + 1, x_0 = 1$
11. $f(x) = x^2 + 4x + 4, x_0 = 1$
12. $f(x) = x^2 + 9, x_0 = 2$
13. $f(x) = 2x^2 + x + 1, x_0 = -\frac{1}{2}$
14. $f(x) = 3x^2 + 2x - 1, x_0 = \frac{2}{3}$

Extra Practice

Extra Practice

Lesson 11-7

(pages 664–669)

Evaluate each expression.

1. $6!$
2. $4!$
3. $\frac{13!}{6!}$
4. $\frac{10!}{3!7!}$
5. $\frac{14!}{4!10!}$
6. $\frac{7!}{2!5!}$
7. $\frac{9!}{8!}$
8. $\frac{10!}{10!0!}$

Expand each power.

9. $(z - 3)^5$
10. $(m + 1)^4$
11. $(x + 6)^4$
12. $(z - y)^2$
13. $(m + n)^5$
14. $(a - b)^4$
15. $(2n + 1)^4$
16. $(3n - 4)^3$
17. $(2n - m)^0$
18. $(4x - a)^4$
19. $(3r - 4s)^5$
20. $\left(\frac{b}{2} - 1\right)^4$

Find the indicated term of each expansion.

21. sixth term of $(x + 3)^8$
22. fourth term of $(x - 2)^7$
23. fifth term of $(a + b)^6$
24. fourth term of $(x - y)^9$
25. sixth term of $(x + 4y)^7$
26. fifth term of $(3x + 5y)^{10}$

Lesson 11-8

(pages 670–673)

Prove that each statement is true for all positive integers.

1. $2 + 4 + 6 + \ldots + 2n = n^2 + n$
2. $1^3 + 3^3 + 5^3 + \ldots + (2n - 1)^3 = n^2(2n^2 - 1)$
3. $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{n(n + 2)} = \frac{n(3n + 5)}{4(n + 1)(n + 2)}$
4. $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \ldots + n(n + 2) = \frac{n(n + 1)(2n + 7)}{6}$
5. $\frac{5}{1 \cdot 2} \cdot \frac{1}{3} + \frac{7}{2 \cdot 3} \cdot \frac{1}{3^2} + \frac{9}{3 \cdot 4} \cdot \frac{1}{3^3} + \ldots + \frac{2n + 3}{n(n + 1)} \cdot \frac{1}{3^n} = 1 - \frac{1}{3^n(n + 1)}$

Find a counterexample for each statement.

6. $n^2 + 2n - 1$ is divisible by 2.
7. $2^n + 3^n$ is prime.
8. $2^{n-1} + n = 2^n + 2 - n$ for all integers $n \geq 2$
9. $3^n - 2n = 3^n - 2^n$ for all integers $n \geq 1$

Lesson 12-1

(pages 684–689)

For Exercises 1–5, state whether the events are *independent* or *dependent*.

1. tossing a penny and rolling a number cube
2. choosing first and second place in an academic competition
3. choosing from three pairs of shoes if only two pairs are available
4. A comedy video and an action video are selected from the video store.
5. The numbers 1–10 are written on pieces of paper and are placed in a hat. Three of them are selected one after the other without replacement.
6. In how many different ways can a 10-question true-false test be answered?
7. A student council has 6 seniors, 5 juniors, and 1 sophomore as members. In how many ways can a 3-member council committee be formed that includes one member of each class?
8. How many license plates of 5 symbols can be made using a letter for the first symbol and digits for the remaining 4 symbols?

Lesson 12-2

(pages 690–695)

Evaluate each expression.

1. $P(3, 2)$
2. $P(5, 2)$
3. $P(10, 6)$
4. $P(4, 3)$
5. $P(12, 2)$
6. $P(7, 2)$
7. $C(8, 6)$
8. $C(20, 17)$
9. $C(9, 4) \cdot C(5, 3)$
10. $C(6, 1) \cdot C(4, 1)$
11. $C(10, 5) \cdot C(8, 4)$
12. $C(7, 6) \cdot C(3, 1)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. choosing a team of 9 players from a group of 20
14. selecting the batting order of 9 players in a baseball game
15. arranging the order of 8 songs on a CD
16. finding the number of 5-card hands that include 4 diamonds and 1 club

Lesson 12-3

(pages 697–702)

A jar contains 3 red, 4 green, and 5 orange marbles. If three marbles are drawn at random and not replaced, find each probability.

1. P(all green)
2. P(1 red, then 2 not red)

Find the odds of an event occurring, given the probability of the event.

3. $\frac{5}{9}$
4. $\frac{4}{8}$
5. $\frac{3}{10}$

Find the probability of an event occurring, given the odds of the event.

6. $\frac{2}{7}$
7. $\frac{6}{13}$
8. $\frac{1}{19}$

The table shows the number of ways to achieve each product when two dice are tossed. Find each probability.

Product	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	30	36
Ways	1	2	2	3	2	4	2	1	2	4	2	1	2	2	2	1	2	1

9. $P(6)$
10. $P(12)$
11. P(not 36)
12. P(not 12)

Lesson 12-4

(pages 703–709)

An octahedral die is rolled twice. The sides are numbered 1–8. Find each probability.

1. P(1, then 8)
2. P(two different numbers)
3. P(8, then any number)

Two cards are drawn from a standard deck of cards. Find each probability if no replacement occurs.

4. P(jack, jack)
5. P(heart, club)
6. P(two diamonds)
7. P(2 of hearts, diamond)
8. P(2 red cards)
9. P(2 black aces)

Determine whether the events are *independent* or *dependent*. Then find the probability.

10. According to the weather reports, the probability of rain on a certain day is 70% in Yellow Falls and 50% in Copper Creek. What is the probability that it will rain in both cities?
11. The odds of winning a carnival game are 1 to 5. What is the probability that a player will win the game three consecutive times?

Extra Practice

Lesson 12-5

(pages 710–715)

An octahedral die is rolled. The sides are numbered 1–8. Find each probability.

1. P(7 or 8)
2. P(less than 4)
3. P(greater than 6)
4. P(not prime)
5. P(odd or prime)
6. P(multiple of 5 or odd)

Ten slips of paper are placed in a container. Each is labeled with a number from 1 through 10. Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. P(1 or 10)
8. P(3 or odd)
9. P(6 or less than 7)

10. Two letters are chosen at random from the word GEESE and two are chosen at random from the word PLEASE. What is the probability that all four letters are Es or none of the letters is an E?
11. Three dice are rolled. What is the probability they all show the same number?
12. Two marbles are simultaneously drawn at random from a bag containing 3 red, 5 blue, and 6 green marbles. Find each probability.
    a. P(at least one red marble)
    b. P(at least one green marble)
    c. P(two marbles of the same color)
    d. P(two marbles of different colors)

Lesson 12-6

(pages 717–723)

Find the mean, median, mode, and standard deviation of each set of data. Round to the nearest hundredth, if necessary.

1. [4, 1, 2, 1, 1]
2. [86, 71, 74, 65, 45, 42, 76]
3. [16, 20, 15, 14, 24, 23, 25, 10, 19]
4. [25.5, 26.7, 20.9, 23.4, 26.8, 24.0, 25.7]
5. [18, 24, 16, 24, 22, 24, 22, 22, 24, 13, 17, 18, 16, 20, 16, 7, 22, 5, 4, 24]
6. [55, 50, 50, 55, 65, 50, 45, 35, 50, 40, 70, 40, 70, 50, 90, 30, 35, 55, 55, 40, 75, 35, 40, 45, 65, 50, 60]
7. [364, 305, 217, 331, 305, 311, 352, 319, 272, 238, 311, 226, 220, 226, 215, 160, 123, 4, 24, 238, 99]

Lesson 12-7

(pages 724–728)

For Exercises 1–4, use the following information.

The diameters of metal fittings made by a machine are normally distributed. The diameters have a mean of 7.5 centimeters and a standard deviation of 0.5 centimeters.

1. What percent of the fittings have diameters between 7.0 and 8.0 centimeters?
2. What percent of the fittings have diameters between 7.5 and 8.0 centimeters?
3. What percent of the fittings have diameters greater than 6.5 centimeters?
4. Of 100 fittings, how many will have a diameter between 6.0 and 8.5 centimeters?

For Exercises 5–7, use the following information.

A college entrance exam was administered at a state university. The scores were normally distributed with a mean of 510, and a standard deviation of 80.

5. What percent would you expect to score above 510?
6. What percent would you expect to score between 430 and 590?
7. What is the probability that a student chosen at random scored between 350 and 670?

Lesson 12-8
(pages 729–733)

HORSES For Exercises 1 and 2, use the following information.

The average lifespan of a horse is 20 years.

1. What is the probability that a randomly selected horse will live more than 25 years?
2. What is the probability that a randomly selected horse will live less than 10 years?

MINIATURE GOLF For Exercises 3 and 4, use the following information.

The probability of reaching in a basket of golf balls at a miniature golf course and picking out a yellow golf ball is 0.25.

3. If 5 golf balls are drawn, what is the probability that at least 2 will be yellow?
4. What is the expected number of yellow golf balls if 8 golf balls are drawn?

Lesson 12-9
(pages 735–739)

Find each probability if a coin is tossed 5 times.

1. P(0 heads)
2. P(exactly 4 heads)
3. P(exactly 3 tails)

Ten percent of a batch of toothpaste is defective. Five tubes of toothpaste are selected at random from this batch. Find each probability.

4. P(0 defective)
5. P(exactly one defective)
6. P(at least three defective)
7. P(less than three defective)

On a 20-question true-false test, you guess at every question. Find each probability.

8. P(all answers correct)
9. P(exactly 10 correct)

Lesson 12-10
(pages 741–744)

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. finding the most often prescribed pain reliever by asking all of the doctors at a hospital
2. taking a poll of the most popular baby girl names this year by studying birth announcements in newspapers from different cities across the country
3. polling people who are leaving a pizza parlor about their favorite restaurant in the city

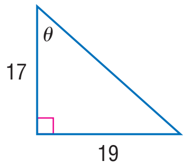
For Exercises 4–6, find the margin of sampling error to the nearest percent.

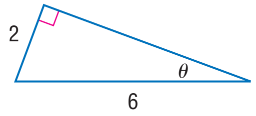
4. $p = 45\%, n = 125$
5. $p = 62\%, n = 240$
6. $p = 24\%, n = 600$
7. A poll conducted on the favorite breakfast choice of students in your school showed that 75% of the 2250 students asked indicated oatmeal as their favorite breakfast.

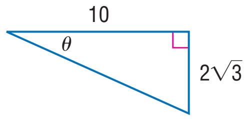
Lesson 13-1

(pages 759–767)

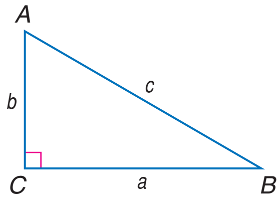
Find the values of the six trigonometric functions for angle θ.

1. 

2. 

3. 

Solve $\triangle ABC$ using the diagram at the right and the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



4. $B = 42°, c = 30$
5. $A = 84°, a = 4$
6. $B = 19°, b = 34$
7. $A = 75°, c = 55$
8. $b = 24, c = 36$
9. $a = 51, c = 115$
10. $\cos B = \frac{2}{5}, a = 12$
11. $\tan A = \frac{3}{2}, b = 22$

Lesson 13-2

(pages 768–774)

Draw an angle with the given measure in standard position.

1. $60°$
2. $250°$
3. $315°$
4. $150°$

Rewrite each degree measure in radians and each radian measure in degrees.

5. $-135°$
6. $-315°$
7. $45°$
8. $80°$
9. $24°$
10. $-54°$
11. $-\pi$
12. $\frac{9\pi}{4}$
13. $\frac{3\pi}{2}$
14. $-\frac{7\pi}{2}$
15. $\frac{9\pi}{10}$
16. $\frac{17\pi}{30}$
17. $\frac{7\pi}{12}$
18. 1
19. $-2\frac{1}{3}$

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

20. $50°$
21. $-75°$
22. $125°$
23. $-400°$
24. $550°$
25. 3π
26. -2π
27. $\frac{2\pi}{3}$
28. $\frac{12\pi}{5}$
29. 0

Lesson 13-3

(pages 776–783)

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. $P(3, -4)$
2. $P(1, \sqrt{3})$
3. $P(0, 24)$
4. $P(-5, -5)$
5. $P(\sqrt{2}, -\sqrt{2})$

Find the exact value of each trigonometric function.

6. $\cos 225°$
7. $\sin\left(-\frac{5\pi}{3}\right)$
8. $\tan \frac{7\pi}{6}$
9. $\tan(-300°)$
10. $\cos \frac{7\pi}{4}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

11. $\cos \theta = -\frac{1}{3}$; Quadrant III
12. $\sec \theta = 2$; Quadrant IV
13. $\sin \theta = \frac{2}{3}$; Quadrant II
14. $\tan \theta = -4$; Quadrant IV
15. $\csc \theta = -5$; Quadrant III
16. $\cot \theta = -2$; Quadrant II
17. $\tan \theta = \frac{1}{3}$; Quadrant III
18. $\cos \theta = \frac{1}{4}$; Quadrant I
19. $\csc \theta = -\frac{5}{2}$; Quadrant IV

Lesson 13-4

(pages 785–792)

Find the area of $\triangle ABC$. Round to the nearest tenth.

1. $a = 11$ m, $b = 13$ m, $C = 31°$
2. $a = 15$ ft, $b = 22$ ft, $C = 90°$
3. $a = 12$ cm, $b = 12$ cm, $C = 50°$

Solve each triangle. Round to the nearest tenth.

4. $A = 18°, B = 37°, a = 15$
5. $A = 60°, C = 25°, c = 3$
6. $B = 40°, C = 32°, b = 10$
7. $B = 10°, C = 23°, c = 8$
8. $A = 12°, B = 60°, b = 5$
9. $A = 35°, C = 45°, a = 30$

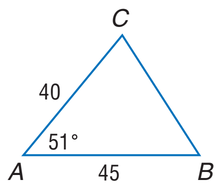
Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle. Round to the nearest tenth.

10. $A = 40°, B = 60°, c = 20$
11. $B = 70°, C = 58°, a = 84$
12. $A = 40°, a = 5, b = 12$
13. $A = 58°, a = 26, b = 29$
14. $A = 38°, B = 63°, c = 15$
15. $A = 150°, a = 6, b = 8$
16. $A = 57°, a = 12, b = 19$
17. $A = 25°, a = 125, b = 150$
18. $C = 98°, a = 64, c = 90$
19. $A = 40°, B = 60°, c = 20$
20. $A = 132°, a = 33, b = 50$
21. $A = 5\ 45°, a = 83, b = 79$

Lesson 13-5

(pages 793–798)

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle.

1.


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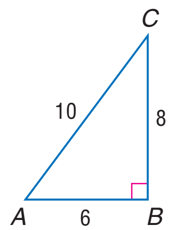
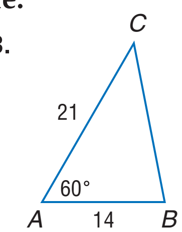


3.


4. $a = 14, b = 15, c = 16$
5. $B = 41°, C = 52°, c = 27$
6. $a = 19, b = 24.3, c = 21.8$
7. $A = 112°, a = 32, c = 20$
8. $b = 8, c = 7, A = 28°$
9. $a = 5, b = 6, c = 7$
10. $C = 25°, a = 12, b = 9$
11. $a = 8, A = 49°, B = 58°$
12. $A = 42°, b = 120, c = 160$
13. $c = 10, A = 35°, C = 65°$
14. $a = 10, b = 16, c = 19$
15. $B = 45°, a = 40, c = 48$
16. $B = 100°, a = 10, c = 8$
17. $A = 40°, B = 45°, c = 4$
18. $A = 20°, b = 100, c = 84$

Lesson 13-6

(pages 799–805)

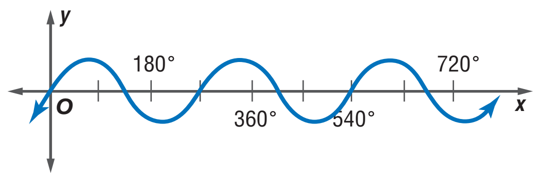
The given point P is located on the unit circle. Find $\sin \theta$ and $\cos \theta$.

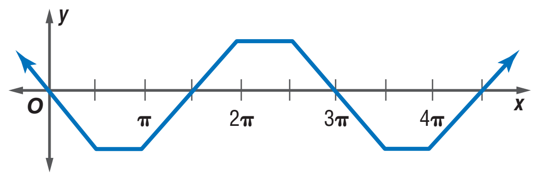
1. $P\left(\frac{4}{5}, \frac{3}{5}\right)$
2. $P\left(\frac{12}{3}, -\frac{5}{13}\right)$
3. $P\left(-\frac{8}{17}, -\frac{15}{17}\right)$
4. $P\left(\frac{3}{7}, \frac{2\sqrt{10}}{7}\right)$
5. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

Find the exact value of each function.

6. $\sin 210°$
7. $\cos 150°$
8. $\cos\ (2135°)$
9. $\cos$
10. $\sin 570°$
11. $\sin 390°$
12. $\sin \frac{4\pi}{3}$
13. $\cos -\frac{7\pi}{3}$
14. $\cos 30° + \cos 60°$
15. $5(\sin 45°)(\cos 45°)$
16. $\frac{\sin 210° + \cos 240°}{3}$
17. $\frac{6 \cos 120° + 4 \sin 150°}{5}$

Determine the period of each function.

18.


19.


Lesson 13–7

(pages 806–811)

Write each equation in the form of an inverse function.

1. $\text{Sin } m + n$
2. $\text{Tan } 45° = 1$
3. $\text{Cos } x = \frac{1}{2}$
4. $\text{Sin } 65° = a$
5. $\text{Tan } 60° = \sqrt{3}$
6. $\text{Sin } x = \frac{\sqrt{2}}{2}$

Solve each equation.

7. $y = \text{Sin}^{-1} -\frac{\sqrt{2}}{2}$
8. $\text{Tan}^{-1}(1) = x$
9. $a = \text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$
10. $\text{Arcsin}(0) = x$
11. $y = \text{Cos}^{-1} \frac{1}{2}$
12. $y = \text{Sin}^{-1}(1)$

Find each value. Round to the nearest hundredth.

13. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$
14. $\text{Sin}^{-1}(-1)$
15. $\cos\left[\text{Arcsin}\left(\frac{\sqrt{2}}{2}\right)\right]$
16. $\tan\left[\text{Sin}^{-1}\left(\frac{15}{13}\right)\right]$
17. $\sin\left[\text{Arccos } \frac{1}{2}\right]$
18. $\sin\left[\text{Arccos}\left(\frac{5}{17}\right)\right]$
19. $\sin\left[\text{Tan}^{-1}\left(\frac{5}{12}\right)\right]$
20. $\tan\left[\text{Arccos } -\left(\frac{\sqrt{3}}{2}\right)\right]$
21. $\sin^{-1}[\text{Cos}^{-1}(1) - 1]$
22. $\text{Cos}^{-1}\left[\tan \frac{\pi}{4}\right]$
23. $\cos\left[\text{Sin}^{-1} \frac{1}{2}\right]$
24. $\sin[\text{Cos}^{-1}(0)]$

Lesson 14-1

(pages 822–828)

Find the amplitude, if it exists, and period of each function. Then graph each function.

1. $y = 2 \cos \theta$
2. $y = \frac{1}{3} \sin \theta$
3. $y = \sin 3\theta$
4. $y = 3 \sec \theta$
5. $y = \sec \frac{1}{3}\theta$
6. $y = 2 \csc \theta$
7. $y = 3 \tan \theta$
8. $y = 3 \sin \frac{2}{3}\theta$
9. $y = 2 \sin \frac{1}{5}\theta$
10. $y = 3 \sin 2\theta$
11. $y = \frac{1}{2} \cos \frac{3}{4}\theta$
12. $y = 5 \csc 3\theta$
13. $y = 2 \cot 6\theta$
14. $y = 2 \csc 6\theta$
15. $y = 3 \tan \frac{1}{3}\theta$

Lesson 14-2

(pages 829–836)

State the phase shift for each function. Then graph the function.

1. $y = \sin(\theta + 60°)$
2. $y = \cos(\theta - 90°)$
3. $y = \tan\left(\theta + \frac{\pi}{2}\right)$
4. $y = \sin \theta + \frac{\pi}{6}$

State the vertical shift and the equation of the midline for each function. Then graph the function.

5. $y = \cos \theta + 3$
6. $y = \sin \theta - 2$
7. $y = \sec \theta + 5$
8. $y = \csc \theta - 6$
9. $y = 2 \sin \theta - 4$
10. $y = \frac{1}{3} \sin \theta + 7$

State the vertical shift, amplitude, period, and phase shift of each function. Then graph the function.

11. $y = 3 \cos[2(\theta + 30°)] + 4$
12. $y = 2 \tan[3(\theta - 60°)] - 2$
13. $y = \frac{1}{2} \sin[4(\theta + 45°)] + 1$
14. $y = \frac{2}{5} \cos[6(\theta + 45°)] - 5$
15. $y = 6 - 2 \sin\left[3\left(\theta + \frac{\pi}{2}\right)\right]$
16. $y = 3 + 3 \cos\left[2\left(\theta - \frac{\pi}{3}\right)\right]$

Lesson 14-3

(pages 837–841)

Find the value of each expression.

1. $\sin \theta$, if $\cos \theta = \frac{4}{5}$; $0° \leq \theta \leq 90°$
2. $\tan \theta$, if $\sin \theta = \frac{1}{2}$; $0° \leq \theta \leq 90°$
3. $\csc \theta$, if $\sin \theta = \frac{3}{4}$; $90° \leq \theta \leq 180°$
4. $\cos \theta$, if $\tan \theta = 24$; $90° \leq \theta \leq 180°$
5. $\sec \theta$, if $\tan \theta = 24$; $90° \leq \theta \leq 180°$
6. $\sin \theta$, if $\cot \theta = -\frac{1}{4}$; $270° \leq \theta \leq 360°$
7. $\tan \theta$, if $\sec \theta = 23$; $90° \leq \theta \leq 180°$
8. $\sin \theta$, if $\cos \theta = \frac{3}{5}$; $270° \leq \theta \leq 360°$
9. $\cos \theta$, if $\sin \theta = -\frac{1}{2}$; $270° \leq \theta \leq 360°$
10. $\csc \theta$, if $\cot \theta = -\frac{1}{4}$; $90° \leq \theta \leq 180°$
11. $\csc \theta$, if $\sec \theta = -\frac{5}{3}$; $180° \leq \theta \leq 270°$
12. $\cos \theta$, if $\tan \theta = 5$; $180° \leq \theta \leq 270°$

Simplify each expression.

13. $\csc^2 \theta - \cot^2 \theta$
14. $\sin \theta \tan \theta \csc \theta$
15. $\tan \theta \csc \theta$
16. $\sec \theta \cot \theta \cos \theta$
17. $\cos \theta (1 - \cos^2 \theta)$
18. $\dfrac{1 - \sin^2 \theta}{\cos^2 \theta}$
19. $\dfrac{\sin^2 \theta + \cos^2 \theta}{\cos^2}$
20. $\dfrac{1 + \tan^2 \theta}{1 + \cot^2 \theta}$
21. $\dfrac{1}{1 + \sin \theta} + \dfrac{1}{1 - \sin \theta}$

Lesson 14-4

(pages 842–846)

Verify that each of the following is an identity.

1. $\sin^2 \theta + \cos^2 \theta + \tan^2 \theta = \sec^2 \theta$
2. $\dfrac{\tan \theta}{\sin \theta} = \sec \theta$
3. $\dfrac{\tan \theta}{\cot \theta} = \tan^2 \theta$
4. $\csc^2 \theta (1 - \cos^2 \theta) = 1$
5. $1 - \cot^4 \theta = 2 \csc^2 \theta - \csc^4 \theta$
6. $\sin^4 \theta - \cos^4 \theta = \sin^2 \theta - \cos^2 \theta$
7. $\sin^2 \theta + \cot^2 \theta \sin^2 \theta = 1$
8. $\dfrac{\cos \theta}{\csc \theta} - \dfrac{\csc \theta}{\sec \theta} = -\dfrac{\cos^3 \theta}{\sin \theta}$
9. $\dfrac{\cos \theta}{\sec \theta} - \dfrac{1 + \cos \theta}{\sec \theta + 1} = 2 \cot^2 \theta$
10. $\dfrac{1 + \cos \theta}{\sin \theta} = \dfrac{\sin \theta}{1 - \cos \theta}$
11. $\sec \theta + \tan \theta = \dfrac{\cos \theta}{1 - \sin \theta}$
12. $\tan \theta + \cot \theta = \csc \theta \sec \theta$
13. $\dfrac{\cot^2 \theta}{1 + \cot^2 \theta} = 1 - \sin^2 \theta$
14. $\dfrac{\tan \theta\, 2 \sin \theta}{\sec \theta} = \dfrac{\sin^3 \theta}{1 + \cos \theta}$
15. $\sin^2 \theta(1 - \cos^2 \theta) = \sin 4\theta$
16. $\sin^2 \theta + \sin^2 \theta \tan^2 \theta = \tan^2 \theta$
17. $\dfrac{\sec \theta - 1}{\sec \theta + 1} + \dfrac{\cos \theta - 1}{\cos \theta + 1} = 0$
18. $\tan^2 \theta(1 - \sin^2 \theta) = \sin^2 \theta$
19. $\tan \theta + \dfrac{\cos \theta}{1 + \sin \theta} = \sec \theta$
20. $\dfrac{\tan \theta}{\sec \theta + 1} = \dfrac{1 - \cos \theta}{\sin \theta}$
21. $\csc \theta - \dfrac{\sin \theta}{1 + \cos \theta} = \cot \theta$

Lesson 14-5

(pages 848–852)

Find the exact value of each expression.

1. $\sin 195°$
2. $\cos 285°$
3. $\sin 255°$
4. $\sin 105°$
5. $\cos 15°$
6. $\sin 15°$
7. $\cos 375°$
8. $\sin 165°$
9. $\sin (-225°)$
10. $\cos (-210°)$
11. $\cos (-225°)$
12. $\sin (-30°)$
13. $\sin 120°$
14. $\sin 225°$
15. $\cos (-30°)$

Verify that each of the following is an identity.

16. $\sin (90° + \theta) = \cos \theta$
17. $\cos (180° - \theta) = -\cos \theta$
18. $\sin (p + \theta) = -\sin \theta$
19. $\sin (\theta + 30°) + \sin (\theta + 60°) = \sqrt{3} + \frac{1}{2}(\sin \theta + \cos \theta)$
20. $\cos (30° - \theta) + \cos (30° + \theta) = \sqrt{3} \cos \theta$

Lesson 14-6

(pages 853–859)

Find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$ for each of the following.

1. $\cos \theta = \frac{7}{25}$; $0 < \theta < 90°$
2. $\sin \theta = \frac{2}{7}$; $0 < \theta < 90°$
3. $\cos \theta = -\frac{1}{8}$; $180° < \theta < 270°$
4. $\sin \theta = -\frac{5}{13}$; $270° < \theta < 360°$
5. $\sin \theta = \frac{\sqrt{35}}{6}$; $0° < \theta < 90°$
6. $\cos \theta = -\frac{17}{18}$; $90° < \theta < 180°$

Find the exact value of each expression by using the half-angle formulas.

7. $\sin 75°$
8. $\cos 75°$
9. $\sin \frac{\pi}{8}$
10. $\cos \frac{13\pi}{12}$
11. $\cos 22.5°$
12. $\cos \frac{\pi}{4}$

Verify that each of the following is an identity.

13. $\frac{\sin 2\theta}{2 \sin^2 \theta} = \cot \theta$
14. $1 + \cos 2\theta = \frac{2}{1 + \tan^2 \theta}$
15. $\csc \theta \sec \theta = 2 \csc 2\theta$
16. $\sin 2\theta (\cot \theta + \tan \theta) = 2$
17. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$
18. $\frac{\csc \theta + \sin \theta}{\csc \theta - \sin \theta} = \frac{1 + \sin 2\theta}{\cos 2\theta}$

Lesson 14-7

(pages 861–866)

Find all the solutions for each equation for $0° \leq \theta < 360°$.

1. $\cos \theta = -\frac{\sqrt{3}}{2}$
2. $\sin 2\theta = -\frac{\sqrt{3}}{2}$
3. $\cos 2\theta = 8 - 15 \sin \theta$
4. $\sin \theta + \cos \theta = 1$
5. $2 \sin^2 \theta + \sin \theta = 0$
6. $\sin 2\theta = \cos \theta$

Solve each equation for all values of θ if θ is measured in radians.

7. $\cos 2\theta \sin \theta = 1$
8. $\sin \frac{\theta}{2} + \cos \frac{\theta}{2} = \sqrt{2}$
9. $\cos 2\theta + 4 \cos \theta = -3$
10. $\sin \frac{\theta}{2} + \cos \theta = 1$
11. $3 \tan^2 \theta - \sqrt{3} \tan \theta = 0$
12. $4 \sin \theta \cos \theta = -\sqrt{3}$

Solve each equation for all values of θ if θ is measured in degrees.

13. $2 \sin^2 \theta - 1 = 0$
14. $\cos \theta - 2 \cos \theta \sin \theta = 0$
15. $\cos 2\theta \sin \theta = 1$
16. $(\tan \theta - 1)(2 \cos \theta + 1) = 0$
17. $2 \cos^2 \theta = 0.5$
18. $\sin \theta \tan \theta - \tan \theta = 0$

Solve each equation for all values of θ.

19. $\tan \theta = 1$
20. $\cos 8\theta = 1$
21. $\sin \theta + 1 = \cos 2\theta$
22. $8 \sin \theta \cos \theta = 2\sqrt{3}$
23. $\cos \theta = 1 + \sin \theta$
24. $2 \cos^2 \theta = \cos \theta$

Mixed Problem Solving

Chapter 1 Equations and Inequalities

(pp. 4–55)

GEOMETRY For Exercises 1 and 2, use the following information.

The formula for the surface area of a sphere is $SA = 4\pi r^2$, and the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. (Lesson 1-1)

1. Find the volume and surface area of a sphere with radius 2 inches. Write your answer in terms of π.
2. Is it possible for a sphere to have the same numerical value for the surface area and volume? If so, find the radius of such a sphere.

3. **CONSTRUCTION** The Birtic family is building a family room on their house. The dimensions of the room are 26 feet by 28 feet. Show how to use the Distributive Property to mentally calculate the area of the room. (Lesson 1-2)

GEOMETRY For Exercises 4–6, use the following information.

The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$. (Lesson 1-2)

4. Use the Distributive Property to rewrite the formula by factoring out the greatest common factor of the two terms.
5. Find the surface area for a cylinder with radius 3 centimeters and height 10 centimeters using both formulas. Leave the answer in terms of π.
6. Which formula do you prefer? Explain your reasoning.

POPULATION For Exercises 7 and 8, use the following information.

In 2004, the population of Bay City was 19,611. For each of the next five years, the population decreased by an average of 715 people per year. (Lesson 1-3)

7. What was the population in 2009?
8. If the population continues to decline at the same rate as from 2004 to 2009, what would you expect the population to be in 2020?

ASTRONOMY For Exercises 9 and 10, use the following information.

The planets in our solar system travel in orbits that are not circular. For example, Pluto's farthest distance from the Sun is 4539 million miles, and its closest distance is 2756 million miles. (Lesson 1-4)

9. What is the average of the two distances ?
10. Write an equation that can be solved to find the minimum and maximum distances from the Sun to Pluto.

HEALTH For Exercises 11 and 12, use the following information.

The National Heart Association recommends that less than 30% of a person's total daily Caloric intake come from fat. One gram of fat yields nine Calories. Jason is a healthy 21-year-old male whose average daily Caloric intake is between 2500 and 3300 Calories. (Lesson 1-5)

11. Write an inequality that represents the suggested fat intake for Jason.
12. What is the greatest suggested fat intake for Jason?

TRAVEL For Exercises 13 and 14, use the following information.

Bonnie is planning a 5-day trip to a convention. She wants to spend no more than \$1000. The plane ticket is \$375, and the hotel is \$85 per night. (Lesson 1-5)

13. Let f represent the cost of food for one day. Write an inequality to represent this situation.
14. Solve the inequality and interpret the solution.

15. **PAINTING** Phil owns and operates a home remodeling business. He estimates that he will need 12–15 gallons of paint for a particular project. If each gallon of paint costs \$18.99, write and solve a compound inequality to determine what the cost c of the paint could be. (Lesson 1-6)

AGRICULTURE **For Exercises 1–3, use the following information.**

The table shows the average prices received by farmers for a bushel of corn. (Lesson 2-1)

Year	Price	Year	Price
1940	\$0.62	1980	\$3.11
1950	\$1.52	1990	\$2.28
1960	\$1.00	2000	\$1.85
1970	\$1.33		

Source: *The World Almanac*

1. Write a relation to represent the data.
2. Graph the relation.
3. Is the relation a function? Explain.

MEASUREMENT **For Exercises 4 and 5, use the following information.**

The equation $y = 0.3937x$ can be used to convert any number of centimeters x to inches y. (Lesson 2-2)

4. Find the number of inches in 100 centimeters.
5. Find the number of centimeters in 12 inches.

POPULATION **For Exercises 6 and 7, use the following information.**

The table shows the growth in the population of Miami, Florida. (Lesson 2-3)

Year	Population	Year	Population
1950	249,276	1990	358,648
1970	334,859	2000	362,437
1980	346,681	2003	376,815

Source: *The World Almanac*

6. Graph the data in the table.
7. Find the average rate of change.

HEALTH **For Exercises 8–10, use the following information.**

In 1985, 39% of people in the United States age 12 and over reported using cigarettes. The percent of people using cigarettes has decreased about 1.7% per year following 1985.
Source: *The World Almanac* (Lesson 2-4)

8. Write an equation that represents how many people use cigarettes in x years.
9. If the percent of people using cigarettes continues to decrease at the same rate, what percent of people would you predict to be using cigarettes in 2005?
10. If the trend continues, when would you predict there to be no people using cigarettes in the U.S.? How accurate is your prediction?

EMPLOYMENT **For Exercises 11–15, use the table that shows unemployment statistics for 1993 to 1999.** (Lesson 2-5)

Year	Number Unemployed	Percent Unemployed
1993	8,940,000	6.9
1994	7,996,000	6.1
1995	7,404,000	5.6
1996	7,236,000	5.4
1997	6,739,000	4.9
1998	6,210,000	4.5
1999	5,880,000	4.2

Source: *The World Almanac*

11. Draw two scatter plots of the data. Let x represent the year.
12. Use two ordered pairs to write an equation for each scatter plot.
13. Compare the two equations.
14. Predict the percent of people that will be unemployed in 2005.
15. In 1999, what was the total number of people in the United States?
16. **EDUCATION** At Madison Elementary, each classroom can have at most 25 students. Draw a graph of a step function that shows the relationship between the number of students x and the number of classrooms y that are needed. (Lesson 2-6)

CRAFTS **For Exercises 17–19, use the following information.**

Priscilla sells stuffed animals at a local craft show. She charges \$10 for the small and \$15 for the large ones. To cover her expenses, she needs to sell at least \$350. (Lesson 2-7)

17. Write an inequality for this situation.
18. Graph the inequality.
19. If she sells 10 small and 15 large animals, will she cover her expenses?

EXERCISE For Exercises 1–4, use the following information.

At Everybody's Gym, you have two options for becoming a member. You can pay \$400 per year or you can pay \$150 per year plus \$5 per visit. (Lesson 3-1)

1. For each option, write an equation that represents the cost of belonging to the gym.
2. Graph the equations. Estimate the break-even point for the gym memberships.
3. Explain what the break-even point means.
4. If you plan to visit the gym at least once per week during the year, which option should you choose?

5. **GEOMETRY** Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines whose equations are $y = 3$, $y = 7$, $y = 2x$, and $y = 2x - 13$. (Lesson 3-2)

EDUCATION For Exercises 6–9, use the following information.

Mr. Gunlikson needs to purchase equipment for his physical education classes. His budget for the year is \$4250. He decides to purchase cross-country ski equipment. He is able to find skis for \$75 per pair and boots for \$40 per pair. He knows that he should buy more boots than skis because the skis are adjustable to several sizes of boots. (Lesson 3-3)

6. Let y be the number of pairs of boots and x be the number of pairs of skis. Write a system of inequalities for this situation. (Remember that the number of pairs of boots and skis must be positive.)
7. Graph the region that shows how many pairs of boots and skis he can buy.
8. Give an example of three different purchases that Mr. Gunlikson can make.
9. Suppose Mr. Gunlikson wants to spend all of the money. What combination of skis and boots should he buy? Explain.

MANUFACTURING For Exercises 10–14, use the following information.

A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one, 1 hour in step two, and produces a profit of \$20. Each pair of indoor shoes requires 1 hour in step one, 3 hours in step two, and produces a profit of \$15. The company has 40 hours of labor per day available for step one and 60 hours available for step two. (Lesson 3-4)

10. Let x represent the number of pairs of outdoor shoes and let y represent the number of indoor shoes that can be produced per day. Write a system of inequalities to represent the number of pairs of outdoor and indoor soccer shoes that can be produced in one day.
11. Draw the graph showing the feasible region.
12. List the coordinates of the vertices of the feasible region.
13. Write a function for the total profit.
14. What is the maximum profit? What is the combination of shoes for this profit?

GEOMETRY For Exercises 15–17, use the following information.

An isosceles trapezoid has shorter base of measure a, longer base of measure c, and congruent legs of measure b. The perimeter of the trapezoid is 58 inches. The average of the bases is 19 inches and the longer base is twice the leg plus 7. (Lesson 3-5)

15. Write a system of three equations that represents this situation.
16. Find the lengths of the sides of the trapezoid.
17. Find the area of the trapezoid.

18. **EDUCATION** The three American universities with the greatest endowments in 2000 were Harvard, Yale, and Stanford. Their combined endowments are \$38.1 billion. Harvard had \$0.1 billion more in endowments than Yale and Stanford together. Stanford's endowments trailed Harvard's by \$10.2 billion. What were the endowments of each of these universities? (Lesson 3-5)

Mixed Problem Solving

AGRICULTURE **For Exercises 1 and 2, use the following information.**

In 2003, the United States produced 63,590,000 metric tons of wheat, 9,034,000 metric tons of rice, and 256,905,000 metric tons of corn. In that same year, Russia produced 34,062,000 metric tons of wheat, 450,000 metric tons of rice, and 2,113,000 metric tons of corn.

Source: *The World Almanac* (Lesson 4-1)

1. Organize the data in two matrices.
2. What are the dimensions of the matrices?

LIFE EXPECTANCY **For Exercises 3–5, use the life expectancy table.** (Lesson 4-2)

Year	1910	1930	1950	1970	1990
Male	48.4	58.1	65.6	67.1	71.8
Female	51.8	61.6	71.1	74.7	78.8

Source: *The World Almanac*

3. Organize all the data in a matrix.
4. Show how to organize the data in two matrices in such a way that you can find the difference between the life expectancies of males and females for the given years. Then find the difference.
5. Does addition of any two of the matrices make sense? Explain.

CRAFTS **For Exercises 6 and 7, use the following information.**

Mrs. Long is selling crocheted items. She sells large afghans for \$60, baby blankets for \$40, doilies for \$25, and pot holders for \$5. She takes the following number of items to the fair: 12 afghans, 25 baby blankets, 45 doilies, and 50 pot holders. (Lesson 4-3)

6. Write an inventory matrix for the number of each item and a cost matrix for the price of each item.
7. Suppose Mrs. Long sells all of the items. Find her total income as a matrix.

GEOMETRY **For Exercises 8–11, use the following information.**

A trapezoid has vertices $T(3, 3)$, $R(-1, 3)$, $A(-2, -4)$, and $P(5, -4)$. (Lesson 4-4)

8. Show how to use a reflection matrix to find the vertices of *TRAP* after a reflection over the x-axis.
9. The area of a trapezoid is found by multiplying one-half the sum of the bases by the height. Find the areas of *TRAP* and $T'R'A'P'$. How do they compare?
10. Show how to use a matrix and scalar multiplication to find the vertices of *TRAP* after a dilation that triples its perimeter.
11. Find the areas of *TRAP* and $T'R'A'P'$ in Exercise 10. How do they compare?

AGRICULTURE **For Exercises 12 and 13, use the following information.**

A farm has a triangular plot defined by the coordinates $\left(-\frac{1}{2}, -\frac{1}{4}\right)$, $\left(\frac{1}{3}, \frac{1}{2}\right)$, and $\left(\frac{2}{3}, -\frac{1}{2}\right)$, where units are in square miles. (Lesson 4-5)

12. Find the area of the region in square miles.
13. One square mile equals 640 acres. To the nearest acre, how many acres are in the triangular plot?

ART **For Exercises 14 and 15, use the following information.**

Small beads sell for \$5.80 per pound, and large beads sell for \$4.60 per pound. Bernadette bought a bag of beads for \$33 that contained 3 times as many pounds of the small beads as the large beads. (Lesson 4-6)

14. Write a system of equations using the information given.
15. How many pounds of small and large beads did Bernadette buy?

MATRICES **For Exercises 16 and 17, use the following information.**

Two 2×2 inverse matrices have a sum of $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$. The value of each entry is no less than -3 and no greater than 2. (Lesson 4-7)

16. Find the two matrices that satisfy the conditions.
17. Explain your method.
18. **CONSTRUCTION** Alan charges \$750 to build a small deck and \$1250 to build a large deck. During the spring and summer, he built 5 more small decks than large decks. If he earned \$11,750, how many of each type of deck did he build? (Lesson 4-8)

PHYSICS **For Exercises 1–3, use the following information.**

A model rocket is shot straight up from the top of a 100-foot building at a velocity of 800 feet per second. (Lesson 5-1)

1. The height $h(t)$ of the model rocket t seconds after firing is given by $h(t) = -16t^2 + at + b$, where a is the initial velocity in feet per second and b is the initial height of the rocket above the ground. Write an equation for the rocket.
2. Find the maximum height of the rocket and the time that the height is reached.
3. Suppose a rocket is fired from the ground (initial height is 0). Find values for a, initial velocity, and t, time, if the rocket reaches a height of 32,000 feet at time t.

RIDES **For Exercises 4 and 5, use the following information.**

An amusement park ride carries riders to the top of a 225-foot tower. The riders then free-fall in their seats until they reach 30 feet above the ground. (Lesson 5-2)

4. Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet, to find how long the riders are in free-fall.
5. Suppose the designer of the ride wants the riders to experience free-fall for 5 seconds before stopping 30 feet above the ground. What should be the height of the tower?

CONSTRUCTION **For Exercises 6 and 7, use the following information.**

Nicole's new house has a small deck that measures 6 feet by 12 feet. She would like to build a larger deck. (Lesson 5-3)

6. By what amount must each dimension be increased to triple the area of the deck?
7. What are the new dimensions of the deck?

NUMBER THEORY **For Exercises 8 and 9, use the following information.**

Two complex conjugate numbers have a sum of 12 and a product of 40. (Lesson 5-4)

8. Find the two numbers.
9. Explain the method you used.

CONSTRUCTION **For Exercises 10 and 11, use the following information.**

A contractor wants to construct a rectangular pool with a length that is twice the width. He plans to build a two-meter-wide walkway around the pool. He wants the area of the walkway to equal the surface area of the pool. (Lesson 5-5)

10. Find the dimensions of the pool to the nearest tenth of a meter.
11. What is the surface area of the pool to the nearest square meter?

PHYSICS **For Exercises 12–14, use the following information.**

A ball is thrown into the air vertically with a velocity of 112 feet per second. The ball was released 6 feet above the ground. The height above the ground t seconds after release is modeled by the equation $h(t) = -16t^2 + 112t + 6$. (Lesson 5-6)

12. When will the ball reach 130 feet?
13. Will the ball ever reach 250 feet? Explain.
14. In how many seconds after its release will the ball hit the ground?

WEATHER **For Exercises 15–17, use the following information.**

The normal monthly high temperatures for Albany, New York, are 21, 24, 34, 46, 58, 67, 72, 70, 61, 50, 40, and 27 degrees Fahrenheit, respectively. **Source:** *The World Almanac* (Lesson 5-7)

15. Suppose January = 1, February = 2, and so on. A graphing calculator gave the following function as a model for the data: $y = -1.5x^2 + 21.2x - 8.5$. Graph the points in the table and the function on the same coordinate plane.
16. Identify the vertex, axis of symmetry, and direction of opening for this function.
17. Discuss how well you think the function models the actual temperature data.

18. **MODELS** John is building a display table for model cars. He wants the perimeter of the table to be 26 feet, but he wants the area of the table to be no more than 30 square feet. What could the width of the table be? (Lesson 5-8)

1. **EDUCATION** In 2002 in the United States, there were 3,034,065 classroom teachers and 48,201,550 students. An average of \$7731 was spent per student. Find the total amount of money spent for students in 2002. Write the answer in both scientific and standard notation. **Source:** *The World Almanac* (Lesson 6-1)

POPULATION For Exercises 2–4, use the following information.

In 2000, the population of Mexico City was 18,131,000, and the population of Bombay was 18,066,000. It is projected that, until the year 2015, the population of Mexico City will increase at the rate of 0.4% per year and the population of Bombay will increase at the rate of 3% per year. **Source:** *The World Almanac* (Lesson 6-2)

2. Let r represent the rate of increase in population for each city. Write a polynomial to represent the population of each city in 2002.
3. Predict the population of each city in 2015.
4. If the projected rates are accurate, in what year will the two cities have approximately the same population?

POPULATION For Exercises 5–8, use the following information.

The table shows the percent of the U.S. population that was foreign-born during various years. The x-values are years since 1900 and the y-values are the percent of the population. **Source:** *The World Almanac* (Lesson 6-3 and 6-4)

U.S. Foreign-Born Population

x	y	x	y
0	13.6	60	5.4
10	14.7	70	4.7
20	13.2	80	6.2
30	11.6	90	8.0
40	8.8	100	10.4
50	6.9		

5. Graph the function.
6. Describe the turning points of the graph and its end behavior.
7. What do the relative maxima and minima represent?
8. If this graph were modeled by a polynomial equation, what is the least degree the equation could have?

GEOMETRY For Exercises 9 and 10, use the following information.

Hero's formula for the area of a triangle is given by $A = \sqrt{s(s - a)(s - b)(s - c)}$, where a, b, and c are the lengths of the sides of the triangle and $s = 0.5(a + b + c)$. (Lesson 6-5)

9. Find the lengths of the sides of the triangle given in this application of Hero's formula: $A = \sqrt{s^4 - 12s^3 + 47s^2 - 60s}$.
10. What type of triangle is this?

GEOMETRY For Exercises 11 and 12, use the following information.

The volume of a rectangular box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always $x + 2$. (Lesson 6-6)

11. What are the width and length of the box?
12. Will the ratio of the dimensions of the box always be the same regardless of the value of x? Explain .

SALES For Exercises 13 and 14, use the following information.

The sales of items related to information technology can be modeled by $S(x) = -1.7x^3 + 18x^2 + 26.4x + 678$, where x is the number of years since 1996 and y is billions of dollars. **Source:** *The World Almanac* (Lesson 6-7)

13. Use synthetic substitution to estimate the sales for 2003 and 2006.
14. Do you think this model is useful in estimating future sales? Explain.
15. **MANUFACTURING** A box measures 12 inches by 16 inches by 18 inches. The manufacturer will increase each dimension of the box by the same number of inches and have a new volume of 5985 cubic inches. How much should be added to each dimension? (Lesson 6-8)
16. **CONSTRUCTION** A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base and the height is 1 more than 3 times the shorter base. What are the dimensions if the area is 4104 square feet? (Lesson 6-9)

Chapter 7 Radical Equations and Inequalities (pp. 382–437)

EMPLOYMENT For Exercises 1 and 2, use the following information.

From 1994 to 1999, the number of employed women and men in the United States, age 16 and over, can be modeled by the following equations, where x is the number of years since 1994 and y is the number of people in thousands. **Source:** *The World Almanac* (Lesson 7-1)

women: $y = 1086.4x + 56{,}610$

men: $y = 999.2x + 66{,}450$

1. Write a function that models the total number of men and women employed in the United States during this time.
2. If f is the function for the number of men, and g is the function for the number of women, what does $(f - g)(x)$ represent?
3. **HEALTH** The average weight of a baby born at a certain hospital is $7\frac{1}{2}$ pounds, and the average length is 19.5 inches. One kilogram is about 2.2 pounds, and 1 centimeter is about 0.3937 inches. Find the average weight in kilograms and the length in centimeters. (Lesson 7-2)

SAFETY For Exercises 4 and 5, use the following information.

The table shows the total stopping distance x, in meters, of a vehicle and the speed y, in meters per second. (Lesson 7-3)

Distance	92	68	49	32	18
Speed	29	25	20	16	11

4. Graph the data in the table.
5. Graph the function $y = 2\sqrt{2x}$ on the same coordinate plane. How well do you think this function models the given data? Explain.
6. **PHYSICS** The speed of sound in a liquid is $s = \sqrt{\frac{B}{d}}$, where B is known as the bulk modulus of the liquid and d is the density of the liquid. For water, $B = 2.1 \cdot 10^9\ \text{N/m}^2$ and $d = 10^3\ \text{kg/m}^3$. Find the speed of sound in water to the nearest meter per second. (Lesson 7-4)

LAW ENFORCEMENT For Exercises 7 and 8, use the following information.

The approximate speed s in miles per hour that a car was traveling if it skidded d feet is given by the formula $s = 5.5\sqrt{kd}$, where k is the coefficient of friction. (Lesson 7-5)

7. For a dry concrete road, $k = 0.8$. If a car skids 110 feet on a dry concrete road, find its speed in miles per hour to the nearest whole number.
8. Another formula using the same variables is $s = 2\sqrt{5kd}$. Compare the results using the two formulas. Explain any variations in the answers.

PHYSICS For Exercises 9–11, use the following information.

Kepler's Third Law of planetary motion states that the square of the orbital period of any planet, in Earth years, is equal to the cube of the planet's distance from the Sun in astronomical units (AU). **Source:** *The World Almanac* (Lesson 7-6)

9. The orbital period of Mercury is 87.97 Earth days. What is Mercury's distance from the Sun in AU?
10. Pluto's period of revolution is 247.66 Earth years. What is Pluto's distance from the Sun?
11. What is Earth's distance from the Sun in AU? Explain your result.

PHYSICS For Exercises 12 and 13, use the following information.

The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared. (Lesson 7-7)

12. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?
Source: *The Guinness Book of Records*
13. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

MANUFACTURING For Exercises 1–3, use the following information.

The volume of a shipping container in the shape of a rectangular prism can be represented by the polynomial $6x^3 + 11x^2 + 4x$, where the height is x. (Lesson 8-1)

1. Find the length and width of the container.
2. Find the ratio of the three dimensions of the container when $x = 2$.
3. Will the ratio of the three dimensions be the same for all values of x?

PHOTOGRAPHY For Exercises 4–6, use the following information.

The formula $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ can be used to determine how far the film should be placed from the lens of a camera. The variable q represents the distance from the lens to the film, f represents the focal length of the lens, and p represents the distance from the object to the lens. (Lesson 8-2)

4. Solve the formula for $\frac{1}{p}$.
5. Write the expression containing f and q as a single rational expression.
6. If a camera has a focal length of 8 centimeters and the lens is 10 centimeters from the film, how far should an object be from the lens so that the picture will be in focus?

PHYSICS For Exercises 7 and 8, use the following information.

The Inverse Square Law states that the relationship between two variables is related to the equation $y = \frac{1}{x^2}$. (Lesson 8-3)

7. Graph $y = \frac{1}{x^2}$.
8. Give the equations of any asymptotes.

PHYSICS For Exercises 9 and 10, use the following information.

The formula for finding the gravitational force between two objects is $F = G\frac{m_A m_B}{d^2}$, where F is the gravitational force between the objects, G is the universal constant, m_A is the mass of the first object, m_B is the mass of the second object, and d is the distance between the centers of the objects. (Lesson 8-4)

9. If the mass of object A is constant, does Newton's formula represent a *direct* or *inverse* variation between the mass of object B and the distance?
10. The value of G is 6.67×10^{-11} N • m^2/kg^2. If two objects each weighing 5 kilograms are placed so that their centers are 0.5 meter apart, what is the gravitational force between the two objects?

EDUCATION For Exercises 11–13, use the table that shows the average number of students per computer in United States public schools for various years. (Lesson 8-5)

Year	Students	Year	Students
1988	32	1996	10
1989	25	1997	7.8
1990	22	1998	6.1
1991	20	1999	5.7
1992	18	2000	5.4
1993	16	2001	5.0
1994	14	2002	4.9
1995	10.5	2003	4.9

Source: *The World Almanac*

11. Let x represent years where 1988 = 1, 1989 = 2, and so on. Let y represent the number of students. Graph the data.
12. What type of function does the graph most closely resemble?
13. Use a graphing calculator to find an equation that models the data.

TRAVEL For Exercises 14 and 15, use the following information.

A trip between two towns takes 4 hours under ideal conditions. The first 150 miles of the trip is on an interstate, and the last 130 miles is on a highway with a speed limit that is 10 miles per hour less than on the interstate. (Lesson 8-6)

14. If x represents the speed limit on the interstate, write expressions for the time spent at that speed and for the time spent on the other highway.
15. Write and solve an equation to find the speed limits on the two highways.

POPULATION **For Exercises 1–4, use the following information.**

In 1950, the world population was about 2.556 billion. By 1980, it had increased to about 4.458 billion. **Source:** *The World Almanac* (Lesson 9-1)

1. Write an exponential function of the form $y = ab^x$ that could be used to model the world population y in billions for 1950 to 1980. Write the equation in terms of x, the number of years since 1950. (Round the value of b to the nearest ten-thousandth.)
2. Suppose the population continued to grow at that rate. Estimate the population in 2000.
3. In 2000, the population of the world was about 6.08 billion. Compare your estimate to the actual population.
4. Use the equation you wrote in Exercise 1 to estimate the world population in the year 2020. How accurate do you think the estimate is? Explain your reasoning.

EARTHQUAKES **For Exercises 5–8, use the following information.**

The table shows the Richter scale that measures earthquake intensity. Column 2 shows the increase in intensity between each number. For example, an earthquake that measures 7 is 10 times more intense than one measuring 6. (Lesson 9-2)

Richter Number x	Increase in Magnitude y
1	1
2	10
3	100
4	1000
5	10,000
6	100,000
7	1,000,000
8	10,000,000

Source: The New York Public Library

5. Graph this function.
6. Write an equation of the form $y = b^{x-c}$ for the function in Exercise 5. (*Hint:* Write the values in the second column as powers of 10 to see a pattern and find the value of c.)
7. Graph the inverse of the function in Exercise 6.
8. Write an equation of the form $y = \log_{10} x + c$ for the function in Exercise 7.

EARTHQUAKES **For Exercises 9 and 10, use the table showing the magnitude of some major earthquakes.** (Lesson 9-3)

Year/Location	Magnitude
1939/Turkey	8.0
1963/Yugoslavia	6.0
1970/Peru	7.8
1988/Armenia	7.0
2004/Morocco	6.4

Source: *The World Almanac*

9. For which two earthquakes was the intensity of one 10 times that of the other? For which two was the intensity of one 100 times that of the other?
10. What would be the magnitude of an earthquake that is 1000 times as intense as the 1963 earthquake in Yugoslavia?

11. Suppose you know that $\log_7 2 \approx 0.3562$ and $\log_7 3 \approx 0.5646$. Describe two different methods that you could use to approximate $\log_7 2.5$. (You can use a calculator, of course.) Then describe how you can check your result. (Lesson 9-4)

WEATHER **For Exercises 12 and 13, use the following information.**

The atmospheric pressure P, in bars, of a given height on Earth is given by using the formula $P = s \cdot e^{-\frac{k}{H}}$. In the formula, s is the surface pressure on Earth, which is approximately 1 bar, h is the altitude for which you want to find the pressure in kilometers, and H is always 7 kilometers. (Lesson 9-5)

12. Find the pressure for 2, 4, and 7 kilometers.
13. What do you notice about the pressure as altitude increases?

AGRICULTURE **For Exercises 14–16, use the following information.**

An equation that models the decline in the number of U.S. farms is $y = 3{,}962{,}520(0.98)^x$, where x is years since 1960 and y is the number of farms. **Source:** *Wall Street Journal* (Lesson 9-6)

14. How can you tell that the number is declining?
15. By what annual rate is the number declining?
16. Predict when the number of farms will be less than 1.5 million.

GEOMETRY For Exercises 1–4, use the following information.

Triangle ABC has vertices $A(2, 1)$, $B(-6, 5)$, and $C(-2, -3)$. (Lesson 10-1)

1. An isosceles triangle has two sides with equal length. Is $\triangle ABC$ isosceles? Explain.
2. An equilateral triangle has three sides of equal length. Is $\triangle ABC$ equilateral? Explain.
3. Triangle EFG is formed by joining the midpoints of the sides of $\triangle ABC$. What type of triangle is $\triangle EFG$? Explain.
4. Describe any relationship between the lengths of the sides of the two triangles.

ENERGY For Exercises 5–8, use the following information.

A parabolic mirror can be used to collect solar energy. The mirrors reflect the rays from the Sun to the focus of the parabola. The latus rectum of a particular mirror is 40 feet long. (Lesson 10-2)

5. Write an equation for the parabola formed by the mirror if the vertex of the mirror is 9.75 feet below the origin.
6. One foot is exactly 0.3048 meter. Rewrite the equation in terms of meters.
7. Graph one of the equations for the mirror.
8. Which equation did you choose to graph? Explain.

COMMUNICATION For Exercises 9–11, use the following information.

The radio tower for KCGM has a circular radius for broadcasting of 65 miles. The radio tower for KVCK has a circular radius for broadcasting of 85 miles. (Lesson 10-3)

9. Let the radio tower for KCGM be located at the origin. Write an equation for the set of points at the maximum broadcast distance from the tower.
10. The radio tower for KVCK is 50 miles south and 15 miles west of the KCGM tower. Let each mile represent one unit. Write an equation for the set of points at the maximum broadcast distance from the KVCK tower.
11. Graph the two equations and show the area where the radio signals overlap.

ASTRONOMY For Exercises 12–14, use the table that shows the closest and farthest distances of Venus and Jupiter from the Sun in millions of miles. (Lesson 10-4)

Planet	Closest	Farthest
Venus	66.8	67.7
Jupiter	460.1	507.4

Source: *The World Almanac*

12. Write an equation for the orbit of each planet, assuming that the center of the orbit is the origin, the center of the Sun is a focus, and the Sun lies on the x-axis.
13. Find the eccentricity for each planet.
14. Which planet has an orbit that is closer to a circle? Explain your reasoning.
15. A comet follows a path that is one branch of a hyperbola. Suppose Earth is the center of the hyperbolic curve and has coordinates (0, 0). Write an equation for the path of the comet if $c = 5{,}225{,}000$ miles and $a = 2{,}500{,}000$ miles. Let the x-axis be the transverse axis. (Lesson 10-5)

AVIATION For Exercises 16–18, use the following information.

The path of a military jet during an air show can be modeled by a conic section with equation $24x^2 + 1000y - 31{,}680x - 45{,}600 = 0$, where distances are in feet. (Lesson 10-6)

16. Identify the shape of the path of the jet. Write the equation in standard form.
17. If the jet begins its ascent at (0, 0), what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?
18. What is the maximum height of the jet?

SATELLITES For Exercises 19 and 20, use the following information.

The equations of the orbits of two satellites are $\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1$ and $\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$, where distances are in km and Earth is the center of each curve. (Lesson 10-7)

19. Solve each equation for y.
20. Use a graphing calculator to estimate the intersection points of the two orbits.

Mixed Problem Solving

CLUBS **For Exercises 1 and 2, use the following information.**

A quilting club consists of 9 members. Every week, each member must bring one completed quilt square. (Lesson 11-1)

1. Find the first eight terms of the sequence that describes the total number of squares that have been made after each meeting.
2. One particular quilt measures 72 inches by 84 inches and is being designed with 4-inch squares. After how many meetings will the quilt be complete?

ART **For Exercises 3 and 4, use the following information.**

Alberta is making a beadwork design consisting of rows of colored beads. The first row consists of 10 beads, and each consecutive row will have 15 more beads than the previous row. (Lesson 11-2)

3. Write an equation for the number of beads in the nth row.
4. Find the number of beads in the design if it contains 25 rows.

GAMES **For Exercises 5 and 6, use the following information.**

An audition is held for a TV game show. At the end of each round, one half of the prospective contestants are eliminated from the competition. On a particular day, 524 contestants begin the audition. (Lesson 11-3)

5. Write an equation for finding the number of contestants that are left after n rounds.
6. Using this method, will the number of contestants that are to be eliminated always be a whole number? Explain.

SPORTS **For Exercises 7–9, use the following information.**

Caitlin is training for a marathon (about 26 miles). She begins by running 2 miles. Then, when she runs every other day, she runs one and a half times the distance she ran the time before. (Lesson 11-4)

7. Write the first five terms of a sequence describing her training schedule.
8. When will she exceed 26 miles in one run?
9. When will she have run 100 total miles?

GEOMETRY **For Exercises 10–12, use a square of paper at least 8 inches on a side.** (Lesson 11-5)

10. Let the square be one unit. Cut away one half of the square. Call this piece Term 1. Next, cut away one half of the remaining sheet of paper. Call this piece Term 2. Continue cutting the remaining paper in half and labeling the pieces with a term number as long as possible. List the fractions represented by the pieces.
11. If you could cut squares indefinitely, you would have an infinite series. Find the sum of the series.
12. How does the sum of the series relate to the original square of paper?

BIOLOGY **For Exercises 13–15, use the following information.**

In a particular forest, scientists are interested in how the population of wolves will change over the next two years. One model for animal population is the Verhulst population model, $p_{n+1} = p_n + rp_n(1 - p_n)$, where n represents the number of time periods that have passed, p_n represents the percent of the maximum sustainable population that exists at time n, and r is the growth factor. (Lesson 11-6)

13. To find the population of the wolves after one year, evaluate $p_1 = 0.45 + 1.5(0.45)(1 - 0.45)$.
14. Explain what each number in the expression in Exercise 13 represents.
15. The current population of wolves is 165. Find the new population by multiplying 165 by the value in Exercise 13.

16. **PASCAL'S TRIANGLE** Study the first eight rows of Pascal's triangle. Write the sum of the terms in each row as a list. Make a conjecture about the sums of the rows of Pascal's triangle. (Lesson 11-7)

17. **NUMBER THEORY** Two statements that can be proved using mathematical induction are $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^n} = \frac{1}{2}\left(1 - \frac{1}{3^n}\right)$ and $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^n} = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$.

    Write and prove a conjecture involving $\frac{1}{5}$ that is similar to the statements. (Lesson 11-8)

NUMBER THEORY For Exercises 1–3, use the following information.

According to the Rational Zero Theorem, if $\frac{p}{q}$ is a rational root, then p is a factor of the constant of the polynomial, and q is a factor of the leading coefficient. (Lesson 12-1)

1. What is the maximum number of possible rational roots that you may need to check for the polynomial $3x^4 - 5x^3 + 2x^2 - 7x + 10$? Explain your answer.
2. Why may you not need to check the maximum number of possible roots?
3. Are choosing the numerator and the denominator for a possible rational root independent or dependent events?

4. **GARDENING** A gardener is selecting plants for a special display. There are 15 varieties of pansies from which to choose. The gardener can only use 9 varieties in the display. How many ways can 9 varieties be chosen from the 15 varieties? (Lesson 12-2)

SPEED LIMITS For Exercises 5 and 6, use the following information.

Speed Limit	Number of States
60	1
65	20
70	16
75	13

Source: *The World Almanac*

The table shows the number of states having each maximum speed limit for their rural interstates. (Lesson 12-3)

5. If a state is randomly selected, what is the probability that its speed limit is 75? 60?
6. If a state is randomly selected, what is the probability that its speed limit is 60 or greater?

7. **LOTTERIES** A lottery number for a particular state has seven digits, which can be any digit from 0 to 9. It is advertised that the odds of winning the lottery are 1 to 10,000,000. Is this statement about the odds correct? Explain your reasoning. (Lesson 12-4)

For Exercises 8 and 9, use the table that shows the most popular colors for luxury cars in 2003. (Lesson 12-5)

Color	% of cars	Color	% of cars
gray	23.3	red	3.9
silver	18.8	blue	3.8
wh. metallic	17.8	gold	3.6
white	12.6	lt. blue	3.1
black	10.9	other	2.2

Source: *The World Almanac*

8. If a car is randomly selected, what is the probability that it is gray or silver?
9. In a parking lot of 1000 cars sold in 2003, how many cars would you expect to be white or black?

EDUCATION For Exercises 10–12, use the following information.

The list shows the average scores for each state for the ACT for 2003–2004. (Lesson 12-6)

20.2, 21.3, 21.5, 20.4, 21.6, 20.3, 22.5, 21.5, 17.8, 20.5, 20.0, 21.7, 21.3, 20.3, 21.6, 22.0, 21.6, 20.3, 19.8, 22.6, 20.8, 22.4, 21.4, 22.2, 18.8, 21.5, 21.7, 21.7, 21.2, 22.5, 21.2, 20.1, 22.3, 20.3, 21.2, 21.4, 20.6, 22.5, 21.8, 21.9, 19.3, 21.5, 20.5, 20.3, 21.5, 22.7, 20.9, 22.5, 22.2, 21.4

10. Compare the mean and median of the data.
11. Find the standard deviation of the data. Round to the nearest hundredth.
12. Suppose the state with an average score of 20.0 incorrectly reported the results. The score for the state is actually 22.5. How are the mean and median of the data affected by this data change?

13. **HEALTH** The heights of students at Madison High School are normally distributed with a mean of 66 inches and a standard deviation of 2 inches. Of the 1080 students in the school, how many would you expect to be less than 62 inches tall? (Lesson 12-7)

14. **SURVEY** A poll of 1750 people shows that 78% enjoy travel. Find the margin of the sampling error for the survey. (Lesson 12-9)

CABLE CARS **For Exercises 1 and 2, use the following information.**

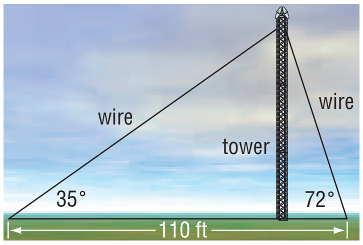
The longest cable car route in the world begins at an altitude of 5379 feet and ends at an altitude of 15,629 feet. The ride is 8-miles long. Source: The Guinness Book of Records (Lesson 13-1)

1. Draw a diagram to represent this situation.
2. To the nearest degree, what is the average angle of elevation of the cable car ride?

RIDES **For Exercises 3 and 4, use the following information.**

In 2000, a gigantic Ferris wheel, the London Eye, opened in England. The wheel has 32 cars evenly spaced around the circumference. (Lesson 13-2)

3. What is the measure, in degrees, of the angle between any two consecutive cars?
4. If a car is located such that the measure in standard position is 260°, what are the measures of one angle with positive measure and one angle with negative measure coterminal with the angle of this car?
5. **BASKETBALL** A person is selected to try to make a shot at a distance of 12 feet from the basket. The formula $R = \frac{V_0^2 \sin 2\theta}{32}$ gives the distance of a basketball shot with an initial velocity of V_0 feet per second at an angle of u with the ground. If the basketball was shot with an initial velocity of 24 feet per second at an angle of 75°, how far will the basketball travel? (Lesson 13-3)
6. **COMMUNICATIONS** A telecommunications tower needs to be supported by two wires. The angle between the ground and the tower on one side must be 35° and the angle between the ground and the second tower must be 72°. The distance between the two wires is 110 feet.



To the nearest foot, what should be the lengths of the two wires? (Lesson 13-4)

SURVEYING **For Exercises 7 and 8, use the following information.**

A triangular plot of farm land measures 0.9 by 0.5 by 1.25 miles. (Lesson 13-5)

7. If the plot of land is fenced on the border, what will be the angles at which the fences of the three sides meet? Round to the nearest degree.
8. What is the area of the plot of land? (*Hint:* Use the area formula in Lesson 13-4.)
9. **WEATHER** The monthly normal temperatures, in degrees Fahrenheit, for New York City are given in the table. January is assigned a value of 1, February a value of 2, and so on. (Lesson 13-6)

Month	Temperature	Month	Temperature
1	32	7	77
2	34	8	76
3	42	9	68
4	53	10	58
5	63	11	48
6	72	12	37

A trigonometric model for the temperature T in degrees Fahrenheit of New York City at t months is given by $T = 22.5 \sin \left(\frac{\pi}{6x} - 2.25\right) + 54.3$. A quadratic model for the same situation is $T = -1.34x^2 + 18.84x + 5$. Which model do you think best fits the data? Explain your reasoning.

PHYSICS **For Exercises 10–12, use the following information.**

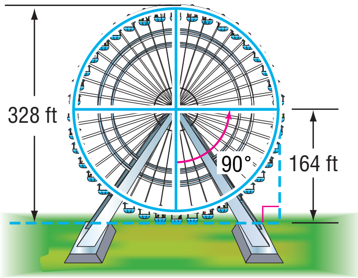
When light passes from one substance to another, it may be reflected and refracted. Snell's law can be used to find the angle of refraction as a beam of light passes from one substance to another. One form of the formula for Snell's law is $n_1 \sin \theta_1 = n_2 \sin \theta_2$, where n_1 and n_2 are the indices of refraction for the two substances and θ_1 and θ_2 are the angles of the light rays passing through the two substances. (Lesson 13-7)

10. Solve the equation for $\sin \theta_1$.
11. Write an equation in the form of an inverse function that allows you to find θ_1.
12. If a light beam in air with index of refraction of 1.00 hits a diamond with index of 2.42 at an angle of 30°, find the angle of refraction.

1. **TIDES** The world's record for the hightest tide is held by the Minas Basin in Nova Scotia, Canada, with a tidal range of 54.6 feet. A tide is at equilibrium when it is at its normal level halfway between its highest and lowest points. Write an equation to represent the height h of the tide. Assume that the tide is at equilibrium at $t = 0$, that the high tide is beginning, and that the tide completes one cycle in 12 hours. (Lesson 14-1)

RIDES For Exercises 2 and 3, use the following information.

The Cosmoclock 21 is a huge Ferris wheel in Yokohama City, Japan. The diameter is 328 feet. Suppose that a rider enters the ride at 0 feet and then rotates in 90° increments counterclockwise. The table shows the angle measures of rotation and the height above the ground of the rider. (Lesson 14-2)



Angle	Height	Angle	Height
0°	0	450°	164
90°	164	540°	328
180°	328	630°	164
270°	164	720°	0
360°	0		

2. A function that models the data is $y = 164 \cdot (\sin (x - 90°)) + 164$. Identify the vertical shift, amplitude, period, and phase shift of the graph.

3. Write an equation using the sine that models the position of a rider on the Vienna Giant Ferris Wheel in Vienna, Austria, with a diameter of 200 feet. Check your equation by plotting the points and the equation with a graphing calculator.

4. **TRIGONOMETRY** Using the exact values for the sine and cosine functions, show that the identity $\cos^2 \theta + \sin^2 \theta = 1$ is true for angles of measure 30°, 45°, 60°, 90°, and 180°. (Lesson 14-3)

5. **ROCKETS** In the formula $h = \frac{v^2 \sin^2 \theta}{2g} = h$ is the maximum height reached by a rocket, θ is the angle between the ground and the initial path of the object, v is the rocket's initial velocity, and g is the acceleration due to gravity. Verify the identity $\frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \cos^2 \theta}{2g \cot^2 \theta}$. (Lesson 14-4)

WEATHER For Exercises 6 and 7, use the following information.

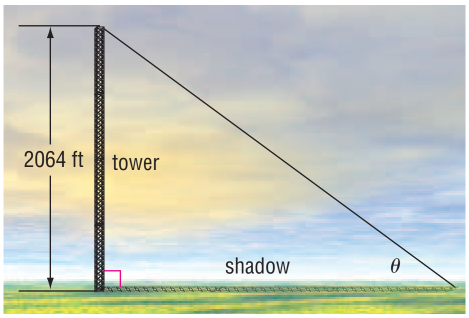
The monthly high temperatures for Minneapolis, Minnesota, can be modeled by the equation $y = 31.65 \sin \left(\frac{\pi}{6x} - 2.09\right) + 52.35$, where the months x are January = 1, February = 2, and so on. The monthly low temperatures for Minneapolis can be modeled by the equation $y = 30.15 \sin \left(\frac{\pi}{6x} - 2.09\right) + 32.95$. (Lesson 14-5)

6. Write a new function by adding the expressions on the right side of each equation and dividing the result by 2.

7. What is the meaning of the function you wrote in Exercise 6?

8. Begin with one of the Pythagorean Identities. Perform equivalent operations on each side to create a new trigonometric identity. Then show that the identity is true. (Lesson 14-6)

9. **TELEVISION** The tallest structure in the world is a television transmitting tower located near Fargo, North Dakota, with a height of 2064 feet.



What is the measure of θ if the length of the shadow is 1 mile? **Source:** *The Guinness Book of Records* (Lesson 14-7)